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Redistribution and Internalization:

The Many-Person Ramsey Tax Rule Revisited*

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ABSTRACT

The paper studies the trade-off between efficiency and equity objectives within a model of commodity taxation. It derives two formulations for a many-person Ramsey tax rule in the presence of externalities. The first tax rule reveals that the aggregate compensated decrease in the demand for a taxed good should be the larger, (i) the more luxurious the good and (ii) the stronger complements the taxed and the polluting good are. The second tax rule shows that the standard many-person Ramsey rule holds for the non-environmental part of a commodity tax, provided that the consumption of the polluting good is already subject to a second-best optimal internalization tax.

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1 Introduction

In recent years it has been recognized that environmental taxation has two objectives. Internalizing externalities allows the environment to be improved while raising tax revenues may allow the excess burden of other taxes to be reduced. Recent papers on the so-called double-dividend hypothesis (cf. e.g. Bovenberg and de Mooij 1994, Goulder 1995, and Schöb 1996) analyse how these two objectives are interrelated.

The literature on the public finance aspects of environmental taxation, however, focus on the efficiency aspect of taxation only. What is largely neglected is the consideration of redistributional objectives. Since Diamond and Mirrlees' (1971) theory of optimal commodity taxation, which encompasses in its most general form both efficiency and equity concerns, it has been frequently pointed out that equity considerations may change the structure of optimal taxes significantly. In particular, it has been shown that differentiated commodity taxes should be used to supplement the income tax as a redistributive device (cf. Atkinson and Stiglitz 1980). Numerical calculations by, e.g. Kaiser, Wiegard and Zimmermann (1990) show that, for an economy where uniform commodity taxes are optimal when efficiency is the sole concern, considering equity drastically changes the optimal tax rates. They show that taxes on goods like food, electricity or rents fall, and may even become negative, when society's inequality aversion increases, while taxes on goods like restaurant meals, transport and communication or personal equipment rise. Although these calculations are only illustrative, they strongly emphasize the importance of distributional objectives in determining optimal commodity taxes.

In an economy with externalities, distributional considerations also affect the optimal environmental tax in two ways. Firstly, many empirical studies, at least those reviewed by Smith (1992) and Harrison (1994), indicate that environmental taxes may have a regressive nature because some environmentally harmful goods are largely consumed by low-income persons. In this case, the presence of redistributional objectives might lower the level of taxes on environmentally harmful commodities. Secondly, distributional considerations also influence the valuation of environmental damage. While the physical incidence of pollution is typically higher in the low-income groups, e.g. due to badly situated housing, well-off people tend to put a higher value on environmental quality (cf. Harrison 1994). To adjust for redistributional objectives individual willingness to pay for environmental quality, summed up to derive the environmental damage, has to be weighted by the social weight given to the individuals in the social welfare function. The stronger society's inequality aversion, the more heavily weighted are the valuations of the poor and, ceteris paribus, environmental taxes should therefore be the larger the more pollution affects the poor.

Following Sandmo (1996), it therefore has to be admitted that although simple policy decision rules in environmental policy – derived by ignoring distributional considerations – may be favourable, there are often good reasons to diverge from simple policy recommendations. Such divergence is necessary to achieve other objectives such as the redistribution of income and the mitigation of strongly regressive influences of environmental policy.

The aim of the present paper is to take account of these objectives by incorporating redistributional concerns into the analysis of second-best pollution taxation. While the seminal paper by Sandmo (1975) analyses an optimal redistributive tax system – though abstracting from the possibility of income taxation – most succeeding papers on second-best environmental taxes ignore distributional aspects. Only recently, redistributional aspects have been considered again. Mayeres and Proost (1995) discuss green tax reforms, while Johansson (1995) concentrates on the externality based part of an optimal commodity tax. In his analysis redistributional concerns arise, however, only from individuals' different valuation of the environmental damage. Sheshinski (1995) argues that personalized environmental taxes can be used to affect redistribution.[1] Nevertheless, the trade-off between efficiency and equity still remains, in our view, an open issue in environmental tax analysis.

We therefore apply an optimal commodity taxation model with linear income taxation, in which distributional concerns arise from differences in the income earning abilities, to derive optimal tax rules in the presence of environmental externalities. We extend Diamond's (1975) many-person Ramsey tax rule by allowing for environmental externalities, which arise from the consumption of an environmentally harmful good. By presenting two alternative formulations of the many-person Ramsey tax rule we show how environmental externalities influence the condition for the optimal tax structure. The first formulation reveals that the compensated change in the demand for a taxed good resulting from a small equiproportional increase in all tax rates depends on both the so-called distributional characteristics of a taxed good and the complementarity relationship between the taxed good and the polluting good. As Sandmo (1975) shows, the tax on a polluting good can be split into an environmental component and a non-environmental component. Applying this separation in the way shown by Orosel and Schöb (1996), we derive a second formulation of the many-person Ramsey tax rule which indicates that the standard many-person Ramsey rule still applies for the non-environmental part of a commodity tax.

The paper is organized as follows. Section 2 presents the model and calculates the first-order conditions for the optimal commodity tax system. The two alternative formulations of the many-person Ramsey tax rule in the presence of externalities are then derived and discussed in Section 3. Section 4 concludes.

[1] In addition, Mayeres and Proost (1997) consider a similar model to that of in this paper to study the optimal taxation of congestion type of externalities. Their emphasis is, however, considerably different from ours.

2 The model

Consider a closed economy with *H* households with identical preferences but different income earning abilities. There are two private consumption goods *c* and *d*, a public good *G* and labour ℓ. The private consumption good *c* is clean, i.e. its consumption has no external effect, whereas the private good *d* is dirty, i.e. its consumption creates negative external effects which cause the environmental quality *E* to deteriorate. The quantities demanded or supplied by household *h* are denoted by $\omega^h \equiv \frac{\partial v^h / \partial E}{\lambda^h}$, the aggregate quantities of the consumption goods are denoted by X_c and X_d, respectively.

There is a linear technology for the production of the private goods and the public good, with labour being the only input. Assuming perfect competition, we can choose units for all goods such that all producer prices are equal to one. As labour productivity differs between households, we denote the marginal productivity of each household's labour by p_ℓ^h. For the normalization chosen, p_ℓ^h also represents the wage rate for household *h*. The production possibilities are described by

$$\sum_h p_\ell^h x_\ell^h = X_c + X_d + G. \tag{1}$$

The government provides the public good *G* and gives to each household a uniform lump-sum subsidy *T* (which might be negative). To finance its expenditures for a given amount of the public good, the government can levy taxes on the private commodities.[2] The government's budget constraint is therefore given by:

$$G + HT = t_c X_c + t_d X_d, \tag{2}$$

where t_c and t_d denote the commodity taxes on the clean good and the dirty good, respectively.

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[2] The question of normalization has caused some confusion about whether a first-best Pigovian tax would exacerbate rather than reduce the deadweight loss of commodity taxes. For a clarification see Fullerton (1997) and Schöb (1997).

Environmental quality deteriorates due to polluting production or consumption. As the main emphasis of this paper is on the interaction of optimal environmental taxes with other forms of taxation, we restrict our analysis to the case of environmental externalities which are proportional to the quantity of a polluting commodity produced or consumed.[3] The environmental quality is thus a decreasing function of the aggregate quantity of the dirty good X_d produced and consumed, i.e.

$$E = e(X_d),\; e' = de/d(X_d) < 0. \tag{3}$$

The preferences of household *h* with respect to both the clean and dirty private good, leisure x_0^h, the public good *G*, and the environmental quality *E*, can be represented by a twice continuously differentiable, strictly quasi-concave utility function

$$U^h = u(x_0^h, x_c^h, x_d^h, G, E), \tag{4}$$

with $u_i > 0,\, i = 0, c, d, G, E,$ denoting the marginal utility of good *i*. The time endowment is normalized to one, hence $x_0^h + x_\ell^h = 1$. The budget constraint of the household is given by

$$(1+t_c)x_c^h + (1+t_d)x_d^h = p_\ell^h x_\ell^h + T, \tag{5}$$

where *T* denotes a lump-sum transfer (which might be negative) granted to each household uniformally. As households differ in their earning abilities, represented by differences in the wage rate, households will also differ in their consumption patterns. When consuming the dirty good, the single household does not take account of the negative effect of its consumption on the environmental quality.

The benevolent government maximizes social welfare, which is represented by a Bergson-Samuelson welfare function

[3] If pollution depends on inputs like energy rather than output, the production efficiency theorem of Diamond and Mirrlees (1971) would imply that the tax rate on the polluting input would consist of an externality-targeted component only, wheras distributive impacts would be taken care of by income and commodity taxes.

$$W = W(v^1(t_c, t_d, T, G, E), v^2(t_c, t_d, T, G, E), \ldots, v^H(t_c, t_d, T, G, E)), \tag{6}$$

subject to its budget constraint (2). The term v^h refers to the indirect utility function of household *h*. Using the indirect utility function implies that the government maximizes social welfare taking account of the individual optimization behaviour. Given the amount of the public good *G*, the government can influence private utility, and hence social welfare, in three ways. First, it can determine private income by varying the lump-sum transfer. Second, it can determine private consumption by imposing commodity taxes. Third, it can determine the environmental quality *E* by imposing a particular environmental tax on the dirty good.[4]

The Lagrangean of government's maximization problem is therefore

$$\mathcal{L} = W(v^1, v^2, \ldots, v^H) + \mu\left[t_c X_c + t_d X_d - G - HT\right]. \tag{7}$$

Denoting the private marginal valuation of income (the Lagrange multiplier of the individual household's optimization problem) by l^h, and using Roy's identity, the first-order conditions are as follows:

$$\frac{\partial L}{\partial t_c} = -\sum_h \frac{\partial W}{\partial v^h} \lambda^h x_c^h + \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial E} \frac{\partial E}{\partial t_c} + \mu\left[X_c + \sum_i t_i \frac{\partial X_i}{\partial t_c} + \sum_i t_i \frac{\partial X_i}{\partial E} \frac{\partial E}{\partial t_c}\right] = 0, \tag{8}$$

$$\frac{\partial L}{\partial t_d} = -\sum_h \frac{\partial W}{\partial v^h} \lambda^h x_d^h + \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial E} \frac{\partial E}{\partial t_d} + \mu\left[X_d + \sum_i t_i \frac{\partial X_i}{\partial t_d} + \sum_i t_i \frac{\partial X_i}{\partial E} \frac{\partial E}{\partial t_d}\right] = 0, \tag{9}$$

$$\frac{\partial L}{\partial T} = \sum_h \frac{\partial W}{\partial v^h} \lambda^h + \sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial E} \frac{\partial E}{\partial T} + \mu\left[-H + \sum_i t_i \frac{\partial X_i}{\partial T} + \sum_i t_i \frac{\partial X_i}{\partial E} \frac{\partial E}{\partial T}\right] = 0. \tag{10}$$

The derivative of *E* with respect to a parameter Z, $Z = t_c, t_d, T$, can be calculated by total differentiation of equation (3):

[4] The more general case where the government also maximizes with respect to the public good provision is analysed in Pirttilä (1996) and Schöb (1995). As we focus on the many-person Ramsey tax rule, however, we do not reproduce these results here.

$$\frac{dE}{dZ}=\frac{e'\sum_h \frac{\partial x_d^h}{\partial Z}}{1-e'\sum_h \frac{\partial x_d^h}{\partial E}}\equiv \sigma e'\sum_h \frac{\partial x_d^h}{\partial Z}, \tag{11}$$

where $\sigma(>0)$ denotes the *environmental feedback* effect. The environmental feedback effect takes account of the fact that the quality of the environment may influence the demand for the dirty good. If a cleaner environment increases the consumption of the dirty good, σ becomes smaller than unity. Peak load pricing e.g. will reduce traffic jams during the rush hour. Less traffic, however, will encourage more traffic.[5] To simplify the analysis, we assume that the utility from the consumption of the private goods *c*, *d* and leisure is weakly separable from the utility from the environment *E*. This implies the assumption that the environmental quality has the same physical impact on all household, independently of their earning abilities and their consumption pattern. In this case, σ reduces to unity and the demand for the dirty good becomes independent of the environmental quality.[6]

To derive optimal tax rules it is convenient to introduce two well-established definitions. First, we define the *gross social marginal valuation* of household *h*'s income, measured in terms of government's revenue by

$$\beta^h \equiv \frac{\partial W}{\partial v^h}\frac{\lambda^h}{\mu}. \tag{12}$$

If the government is interested in redistributing income from high ability households to low ability households, the social welfare function (6) will be strictly quasi-concave, i.e. $\partial W/\partial v^h$ is larger the lower v^h. As private utility is also strictly quasi-concave, λ^h decreases in utility. Hence, β^h is negatively correlated with the earning ability and the household's utility level, respectively.

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5 Stability is guaranteed as long as the denominator of equation (11) is positive (cf. Schöb 1995, p. 118).

6 The separability assumption is not essential for our results. Where appropriate, we briefly refer to the implications a relaxation of this assumption has on the results. The general results without restricting preferences are available from the authors on request.

The marginal willingness to pay for environmental quality is defined as

$$\omega^h \equiv \frac{\partial v^h / \partial E}{\lambda^h}. \tag{13}$$

The individual evaluation of the additional environmental damage may differ between individuals as the marginal valuation of the environment normally does not change proportionately with the marginal utility of income λ^h. Applying the separability assumption and using the definitions (12) and (13), the first-order conditions can be rewritten as:

$$\frac{\partial L}{\partial t_c} = -\sum_h \beta^h x_c^h + \sum_h \beta^h \omega^h e' \frac{\partial X_d}{\partial t_c} + \left[X_c + \sum_i t_i \frac{\partial X_i}{\partial t_c} \right] = 0\,, \tag{8a}$$

$$\frac{\partial L}{\partial t_d} = -\sum_h \beta^h x_d^h + \sum_h \beta^h \omega^h e' \frac{\partial X_d}{\partial t_d} + \left[X_d + \sum_i t_i \frac{\partial X_i}{\partial t_d} \right] = 0\,, \tag{9a}$$

$$\frac{\partial L}{\partial T} = \sum_h \beta^h + \sum_h \beta^h \omega^h e' \frac{\partial X_d}{\partial T} + \left[-H + \sum_i t_i \frac{\partial X_i}{\partial T} \right] = 0\,. \tag{10a}$$

By using Cramer's rule, we can solve equations (8a) and (9a) for the optimal commodity tax rate of the clean and the dirty good, respectively. Denoting the determinant of the Jacobian matrix as $|J|$, the optimal tax formulas are:

$$t_c = \frac{1}{|J|} \sum_h \left(\beta^h - 1 \right) \left(x_c^h \frac{\partial X_d}{\partial t_d} - x_d^h \frac{\partial X_d}{\partial t_c} \right), \tag{14}$$

$$t_d = \frac{1}{|J|} \sum_h \left(\beta^h - 1 \right) \left(x_d^h \frac{\partial X_c}{\partial t_c} - x_c^h \frac{\partial X_c}{\partial t_d} \right) - \sum_h \beta^h \omega^h e'\,. \tag{15}$$

Equations (14) and (15) restate the result originally derived by Sandmo (1975). The external effect does not enter the tax formula for the clean good. It only enters the optimal tax formula for the dirty good additively. This environmental component of the optimal tax on the dirty good may be considered as the price the consumer of the dirty good has to pay in a second-best world in order to completely internalize the external effect.[7]

To see this, consider the following thought experiment which is related to the interpretation of the *second-best internalization tax* presented in Orosel and Schöb (1996) for an economy with identical households. We abstract from all other taxes and focus on the environmental tax component alone which we define as t_d^E. Assume that a household *k* receives an additional marginal unit of exogenous income Y^k. In the household optimum, the household's utility increases by λ^k, independently of how it spends the additional income. Hence, without loss of generality, we assume that the household increases the consumption of *d* only, i.e. by $1/(1+t_d^E)$. The effect of a marginal increase in household *k*'s income on social welfare is therefore (measured in units of public revenues)

$$\frac{dW/dY^k}{\mu} = \frac{\partial W}{\partial v^k}\frac{\lambda^k}{\mu} + \sum_h \frac{1}{\mu}\frac{\partial W}{\partial v^h}\frac{\partial v^h}{\partial E}\varepsilon' \frac{1}{1+t_d^E} + \frac{t_d^E}{1+t_d^E} = \beta^k + \left(\sum_h \beta^h \omega^h \varepsilon' + t_d^E\right)\frac{1}{1+t_d^E}. \quad (16)$$

The first term of the right-hand side denotes the increase in the gross social marginal valuation of household *k*'s private utility β^k (cf. equation (12)). The second term denotes the social marginal external effect imposed on all households by the additional consumption of the dirty good *d* (cf. equation (13)). The last term shows the increase in public revenues from the internalization tax imposed on the dirty good *d*. (It is assumed that additional tax revenues are used to increase public good provision.)

Full internalization requires that, from the viewpoint of society, the *social marginal utility* of the private consumption of the dirty good, i.e. the gross social marginal valuation β^k, should

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[7] Without assuming separability, the environmental component of t_d also contains a third term which takes account of the environmental feedback influence σ, and the impact a change in the environmental quality has on commodity tax revenues. If more pollution leads to a fall in government's tax revenues, the optimal level of the externality is lower than if this were not the case. This increases the magnitude of the externality-based component in t_d.

be equal to *social marginal welfare* of consuming the dirty good: $\beta^k = dW/dY^k/\mu$. Hence, the external effect is exactly internalized if and only if the tax on the dirty good is equal to

$$t_d^E = -\sum_h \beta^h \omega^h e'. \tag{17}$$

which forms the environmental component of t_d in equation (15). The term $\omega^h e'$ denotes the marginal willingness to pay for a reduction in emissions times the amount of emissions caused by an marginal increase in the dirty good consumption. In order to derive the social evaluation of pollution, the household's marginal willingness to pay has to be weighted with the social weight β^h given to the household.

3 Rules for optimal government policy

3.1 The many-person Ramsey tax rule with externalities

Diamond (1975) presents a procedure for interpreting commodity taxation rules when income can be taxed on a linear scale. This section refers to Diamond's approach to deriving a many-person Ramsey tax rule, and demonstrates how his model has to be modified to allow for the presence of externalities. Therefore, we first redefine the *net social marginal valuation* of household *h*'s income, denoted by γ^h, by taking into account the influence private consumption has on the external effect:

$$\gamma^h = \beta^h + \sum_i t_i \frac{\partial x_i^h}{\partial T} + \sum_k \beta^k \omega^k e' \frac{\partial x_d^h}{\partial T} \tag{18}$$

Definition (18) is identical with Diamond's definition (see his equation (6)), except for the last term of the right-hand side.[8] The *net* social marginal valuation of household *h*'s income includes, first of all, the *gross* marginal social valuation of income β^h which represents the

[8] Note, however, that Diamond (1975) measures the net social marginal valuation in terms of private income while the measure presented here is in units of public revenues.

social evaluation of the marginal utility household h derives from a marginal increase in income. However, the social value of an extra income to household h also depends on the influence the additional income has on tax revenues. This effect is captured by the second term of the right-hand side of equation (18). If the extra income increases the demand for taxed goods by household h, tax revenues also increase and may be used e.g. to increase the provision of the public good. In this case, the net social marginal valuation exceeds the gross social valuation of income.

In the presence of externalities, the net social valuation of income also depends on the impact the additional income has on the environmental quality.[9] This effect is covered by the last term on the right-hand side of equation (18). If the extra income increases the household's consumption of the dirty good, the value of the additional damage caused by it to all members of the economy has to be deducted from the social valuation of income. If the dirty good is a normal good, the externality-augmented net social valuation of income therefore will be lower than Diamond's (1975) definition suggests.

Note that, while the gross social valuation of income is larger for low-income households than for well-off households, this is not necessarily true for the net measure, because additional income may lead to larger changes in the demand for the taxed commodities among the high-income households, in which case γ^h would increase. It is also hard to deduce whether the magnitude of externality-encompassing term is greater or smaller for worse-off households and, accordingly, in which direction the differences in the net social valuation move.

Using the definition of γ^h, the Slutsky decomposition

$$\frac{\partial x_i^h}{\partial t_j} = s_{ij}^h - x_j^h \frac{\partial x_i^h}{\partial T},$$

[9] Without assuming separability, the impact of externalities on γ also depends on the tax revenue effect of a change in the environmental quality (cf. footnote 6).

where s_i^h denotes the compensated (cross-) price effect, and Slutsky-symmetry, the first-order conditions for $t_j, j = c,d$ can be rewritten in the following way:

$$\sum_h \sum_i t_i s_{ij}^h = \sum_h (\gamma^h - 1) x_j^h - \sum_h (\beta^h \omega^h e') \sum_h s_{dj}^h \,, \tag{19}$$

with $j = c,d$. In the absence of externalities, i.e. for $e' = 0$, equation (19) restates Diamond's (1975) result (cf. his equation (7)). The new second term of the right-hand side takes account of the externality. In order to interpret equation (19), however, we will further simplify this condition. Substituting definition (18) into the first-order condition for the lump-sum transfer, equation (10a), we obtain:

$$\sum_h \left(\beta^h + \sum_i t_i \frac{\partial x_i^h}{\partial T} + \sum_k \left(\beta^k \omega^k e' \right) \frac{\partial x_d^h}{\partial T} \right) = H \quad \Leftrightarrow \quad \bar{\gamma} = 1, \tag{20}$$

where $\bar{\gamma}$ denotes the average net social valuation of income over all households. Equation (20) states that in an optimum, the average valuation of a transfer of one unit of money should be equal to its cost, which is equal to unity.

Next, we define the normalized covariance between the net social evaluation of private income and the consumption of good *i*,

$$\Phi_i \equiv \frac{\sum_h g^h x_i^h}{\bar{g} X_i} - 1 \tag{21}$$

(cf. e.g. Atkinson and Stiglitz 1980). The first term of the right-hand side is known as the *distributional characteristic* of good *i* (cf. Feldstein (1972)). If the government is indifferent to which household the extra income is given, all γ^h are identical to $\bar{\gamma}$, and the normalized covariance expression reduces to zero. With inequality aversion, γ^h is larger for low-income households, provided that the additional tax revenues and the externality terms in the definition of γ^h do not have a too strong countervailing effect. This implies that the distributional characteristics of a good, of which low-income households demand a large proportion, takes a value greater than unity, because the relative consumption of that good by household h increases with γ^h. According to Rose and Wiegard (1983), the distributional characteristics of a good i can be interpreted as a measure of society's willingness to pay for a more equal distribution of income.[10]

Substituting the definition of the normalized covariance (20) into equation (19) we obtain:

$$\frac{\sum_h \sum_i t_i s_{ij}^h}{X_j} = \Phi_j - \frac{\sum_h (\beta^h \omega^h e')}{1+t_d} \sum_h \varepsilon_{jd}^h \frac{x_j^h}{X_j}, \tag{22-I}$$

with $j = c,d$. The term ε_{jd}^h denotes the compensated cross-price elasticity of good j with respect to the price of the dirty good d. The left-hand side equals the relative change of the compensated demand if all tax rates change proportionately. To see this, consider the total differential of the compensated demand function $x_j^h(t_c, t_d, \bar{u}), j = c,d$ for a small equiproportionate change of all tax rates, i.e. $dt_i = \alpha t_i, i = c,d$:[11]

$$dx_j^h\Big|_{\bar{u}} = \sum_{i=c,d} s_{ji}^h dt_i = \alpha \sum_{i=c,d} s_{ji}^h t_i . \tag{23}$$

Summing up over all households and dividing by the total demand, we obtain the relative change in aggregate demand

$$\frac{\sum_h dx_j^h\Big|_{\bar{u}}}{X_j} = \alpha \frac{\sum_h \sum_i t_i s_{ij}^h}{X_j}. \tag{24}$$

The left-hand side of equation (22-I) therefore explains how the relative change in the demand of good j due to a small equiproportionate change of all tax rates is determined. To interpret the right-hand side carefully, consider first the case without external effects (i.e. $e' = 0$). In this case, the externality-based term disappears from the first-order condition and

[10] They consider a redistribution scheme where the government redistributes income by taking away a certain fraction of each households consumption of good i and rebating the revenues by giving back each household the average amount collected. They then show that, for such a redistribution scheme, the normalized covariance measures the fraction of total revenues (in units of good i) society could throw away without reducing welfare. Hence, it can be interpreted as a measure of the willingness to pay for a more equal distribution achieved by redistributing good i.

[11] Note that we have assumed separability between consumption and environmental quality.

from the definition of γ^h, respectively. Optimal commodity taxation is determined by the normalized covariance alone.

If, on the other hand, redistribution is not an issue, and there are are no external effects, the normalized covariance term vanishes. In this case, there would be no commodity taxation in the optimum. Tax revenues would be raised by imposing a uniform lump-sum tax $(T < 0)$ only. With inequality aversion, however, the aggregate compensated change in demand of good j should be the smaller, the lower the values of the normalized covariance Φ_j. The normalized covariance rule therefore advises the government to subsidize the consumption of goods which are largely demanded by those people with a large net social marginal valuation of income, i.e. the poor people[12] and discourage the consumption of luxury goods consumed by rich households with low g^h. In this way commodity taxation is used for redistributional purposes.

If external effects are present, but society is not interested in redistribution, the aggregate compensated change in the demand of taxed goods is determined solely by the externality term. This term depends on the compensated elasticity between the taxed good and the dirty good. The compensated own-price elasticity of the dirty good, and hence the aggregate compensated change in the demand for the dirty good, will be negative. This impact arises naturally from the fact that, as the consumption of good d worsens the environmental quality, it is in the society's interest to reduce its consumption. For the clean good, the compensated change in demand should be the smaller, the higher the compensated complementarity relationship between the taxed good and the dirty good (the more negative the compensated elasticity is). Proposition 1 summarizes.

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[12] As mentioned above, this need not necessarily be true. It depends on the tax revenue effect (cf. the interpretation of equation (18)). In what follows, however, we assume that poor people have high values of γ^h while rich people are assumed to have low values.

PROPOSITION 1 (The many-person Ramsey tax rule in the presence of externalities, variant I): If commodity taxes and the lump-sum transfer are set optimally, a small equiproportional increase in all tax rates will cause all compensated commodity demands to change according to their distributional characteristics. In addition, the decline (increase) in the compensated demand for the taxed good will be larger (smaller), the stronger the complementarity relationship between the taxed good and the dirty good.

Note that, with an equiproportional change in tax rates, we change the level of the Pigovian tax component by the same amount as the Ramsey tax component. Hence, the tax on the dirty good increases at a larger rate than the standard many-person Ramsey tax rule suggests in the absence of externalities. Consequently, the demand for all complements will fall at a larger rate while the demand of substitutes will fall at a lower rate. This is the mechanism which makes the complementarity relationship between taxed goods and the dirty good enter the tax rule. A comparison with Sandmo's additivity property shows that Proposition 1 does not imply that goods which are complements to the dirty good should be taxed more strongly. Rather, the conclusion suggests that their consumption is reduced because of the indirect impact the relatively larger change of the tax on the dirty good has on their demand.

This result also shows that, apart from the redistributional characteristics of the particular commodity, the optimal tax rule also depends on redistributional concerns because of the social valuation of environmental damage. The private disutility of a marginal increase in pollution is weighted by the gross social valuation of household *h*'s income β^h. This means that the marginal willingness to pay for the environmental quality by a low-income household has a relatively high impact on the social valuation of the environment and on the externality-based term in the tax rules as well. Moreover, as the magnitude of β^h decreases with the shadow price of public funds μ, we can deduce that the externality-based part in the tax conditions decreases with rising μ. The reason is that, as the burden of public funds rises, it becomes more and more expensive to internalize externalities (or, in other words, to provide the public good 'environmental quality') through tax policy - an intuition presented by Bovenberg and van der Ploeg (1994).

In brief, the modified many-person Ramsey tax rule reveals that the influence of commodity taxation on the demand for taxed goods depends on both redistributional and environmental objectives. If the worse-off households have a relatively large demand for the environmentally harmful good, as some empirical studies suggest (cf. e.g. Smith 1992, and Harrison 1994), then the optimal tax rule proposes that the income distribution part of taxation lowers the tax on the dirty good. This tax would otherwise be high in order to internalize the negative external impact. Therefore, without any further restrictions on consumers' preferences, efficiency and equity considerations cannot be separated when making decisions concerning the level of environmental taxes. The result derived here therefore confirms the statement in the introduction that the distributional concerns cannot be ignored in the study of externalities within public finance models. Only if households' preferences are weakly separable between private consumption goods and leisure and the Engel-curves of commodities are linear, does redistribution have no influence on commodity taxation.[13]

This discussion can be connected to the double-dividend debate. Although the formulation containing the change in the compensated demand does not provide an explicit tax rule, it still implies that if the harmful goods are in a relatively great demand by the low-income households, the non-environmental tax component should be low - or even negative - because, in the absence of externalities, differentiated commodity taxation is only used to influence income distribution, and a reduction in the tax rate of a good is a direct way to increase its compensated demand. One of the implications of the strong form of the double dividend notion (cf. Goulder 1995) is that the tax on the harmful good might be higher than the standard first-best Pigovian rule. Our conclusion of the decreasing influence of the

[13] This has been shown by Deaton (1979).

redistributional part of the tax imposed on the harmful good casts then some further doubt on the possibility to gain a double dividend by environmental taxation.

3.2 An alternative formulation of the many-person Ramsey tax rule in the presence of externalities

Making use of the definition of the internalization tax (17), we can decompose the total tax on the dirty good in the following way:

$$t_d = t_d^R + t_d^E \,, \tag{25}$$

whereby we define t_d^R as the Ramsey tax component of the total tax on the dirty good. Using the same decomposition for the clean good and applying Sandmo's (1975) additivity property (cf. our equation (14)), it is easy to see that the Ramsey tax component equals the total tax, i.e. $t_c = t_c^R$. Following the procedure suggested by Orosel and Schöb (1996), we substitute the decomposition (25) into the first-order conditions given in equations (8a) - (10a). Applying definition (17), this yields:

$$\frac{\partial L}{\partial t_c} = -\sum_h \beta^h x_c^h + \left[X_c + t_c \frac{\partial X_c}{\partial t_c} + t_d^R \frac{\partial X_d}{\partial t_c} \right] = 0 \,, \tag{8b}$$

$$\frac{\partial L}{\partial t_d} = -\sum_h \beta^h x_d^h + \left[X_d + t_c \frac{\partial X_c}{\partial t_d} + t_d^R \frac{\partial X_d}{\partial t_d} \right] = 0 \,, \tag{9b}$$

$$\frac{\partial L}{\partial T} = \sum_h \beta^h + \left[-H + t_c \frac{\partial X_c}{\partial T} + t_d^R \frac{\partial X_d}{\partial T} \right] = 0 \,. \tag{10b}$$

Writing the first-order conditions in this way shows that the first-order conditions for the optimal tax system in the presence of externalities differ from the first-order conditions in the model without externalities only insofar as we have to add an additional condition, i.e. equation (25), and replace t_d by t_d^R. The same substitution can be applied for the definition of the net social marginal valuation of household *h*'s income (equation (18)):

$$\tilde{\gamma}^h = \beta^h + t_c \frac{\partial x_c^h}{\partial T} + t_d^R \frac{\partial x_d^h}{\partial T}, \tag{26}$$

in which the tax on the dirty good is replaced by the Ramsey-component of t_d. We can solve the equation system (8b) - (10b) in the same way as demonstrated above. The conditions characterizing the optimal tax structure can therefore be rewritten in the following way:

$$\frac{\sum_h \sum_i t_i^R s_{ij}^h}{X_j} = \tilde{\Phi}_j, \tag{22-II}$$

with $j = c, d$. These tax rules indicate that the many-person Ramsey tax rule developed by Diamond (1975) applies for the non-environmental part of commodity taxes in the presence of externalities, provided that the net social marginal valuation of income, in the covariance term, is calculated taking the externalities into account. The intuition behind the result is that, while the government can internalize externalities by levying a tax rate t_d^E on the dirty good, the rest of the tax system, i.e. the taxation of the clean good and the Ramsey tax component of the tax on the dirty good, can be determined by the distributional concerns alone, based on the normalized covariance. Proposition 2 summarizes.

PROPOSITION 2 (The many-person Ramsey tax rule in the presence of externalities, variant II): If commodity taxes and the lump-sum transfer are set optimally, a small equiproportional increase in all Ramsey tax components will cause all compensated commodity demands to change according to their distributional characteristics.

Notice that there is a difference in the equiproportional tax change experiment here compared to the one in Section 3.1. In the present section the tax change and the derived tax rule apply to the Ramsey tax component of the dirty tax only. Hence, though the tax rate change for the clean good is the same in both tax rules, the tax rate change for the dirty good is smaller here. As a consequence the complementarity relationship which drove the result presented in

Proposition 1, does not apply to proposition 2 as the environmental tax component is kept fixed in absolute terms.

4 Conclusion

The controversy between efficiency and equity, commonly encountered in tax policy analysis, is largely neglected in the literature on second-best environmental taxes. The aim of the present paper is, therefore, to address the potential trade-off between efficiency and equity by studying commodity taxation in a many-person economy where (i) households differ in their income earning capacities, and (ii) environmental externalities arise from the consumption of a dirty good. The results derived here indicate that, when the government can pursue optimal linear income taxation, the role of the commodity taxation consists of the redistribution of income and the correction of the harmful externality. This is demonstrated by extending Diamond's (1975) many-person Ramsey tax rule to the case with environmental externalities. A first modification reveals that the aggregate compensated change in the demand for a taxed good depends, first of all, on the distributional characteristics of a commodity. Consequently, the government should impose high taxes on goods that are largely consumed by well-off households. However, the many-person Ramsey tax rule in the presence of externalities also shows that this rule has to be adjusted by imposing an additional environmental tax on the dirty good so that the compensated change in the demand of a taxed good due to a small equiproportional change in all tax rates is the smaller, the higher the compensated complementarity between the taxed good and the dirty good. This is because an increase of the environmental tax component reduces the demand for goods which are complements for the dirty good while it increases the demand for all goods which are substitutes.

An alternative modification reveals that the standard many-person Ramsey tax rule still applies for the non-environmental components of the optimal tax system. The reason is that an equiproportional change in all non-environmental tax components, leaving the environmental tax constant, would lead to a change in the compensated demands according to their distributional characteristics. This result therefore indirectly confirms Sandmo's (1975) additivity property for second-best optimal environmental taxes.

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