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# DISTORTIONARY DOMESTIC TAXATION AND PARETO-EFFICIENT INTERNATIONAL TRADE

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# DISTORTIONARY DOMESTIC TAXATION AND PARETO-EFFICIENT INTERNATIONAL TRADE


## Abstract

This paper characterises the domestic tax systems which yield Pareto-efficient outcomes for a two-country world economy in which each country uses distortionary taxes. Such outcomes are compared with the Nash equilibria of the world economy when each country uses its domestic tax system to influence ist terms of trade. In such circumstances, the implementation of domestic tax systems which achieve a globally Pareto-efficient outcome as a Nash equilibrium will be very difficult, for two main reasons: the ability of countries to use tax policy with respect to non-traded goods for protection, and the fact that Pareto-efficient tax structures depend on countries' distributional judgements, which are hard to measure objectively.


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# 1 Introduction

A basic argument for international agreements which require countries to reduce trade taxes, such as the General Agreement on Tariffs and Trade (GATT), is that the use of taxes which distort the pattern of trade results in an outcome which is Pareto-inefficient for the world economy. Since domestic tax policy, by which we mean the taxation and subsidisation of consumption and production within a country, can replicate the effects of trade taxes, Article III of the GATT specifies that countries which reduce or eliminate trade taxes should not use domestic taxation or other measures to achieve the same degree of protection. However, the informational and incentive constraints which make optimal redistributive lump-sum taxes and transfers impossible (Hammond 1979) mean that in practice the tax systems of all countries involve the use of distortionary taxes. By their nature, such distortionary taxes will, in general, affect trade flows between countries. Hence an obvious question raised by the requirement that countries' domestic tax policies should not be used for protectionist purposes is the following: what structure of distortionary domestic taxes and subsidies to finance public good provision and achieve redistributional objectives in each country is compatible with a Pareto-efficient outcome for the world economy, i.e. one in which all possible gains from international trade have been exhausted?

The existing literature does not give a clear answer to this question. Much of it analyses taxes on consumption, production, and trade in the context of single-household economies, and often assumes that revenue is returned to the single household in lump-sum form. Of those contributions which provide part of an answer to the question, Dixit and Norman (1980) and Dixit (1985) incorporate distributional and revenue-raising considerations into the analysis of optimal domestic and trade taxes, while Diewert, Turunen-Red and Woodland (1989) establish that strictly Pareto-improving trade tax reforms exist for a small open economy with many households in which lump-sum taxes cannot be used, but all three focus on a single country rather than the world economy. The literature on tax harmonisation (Keen 1987, 1989, Turunen-Red and Woodland

1990, Lockwood 1997) is motivated by the idea that tax harmonisation between countries may be a Pareto-improving way of removing the indirect protection countries obtain from suitably-chosen domestic taxation, but largely neglects revenue-raising and distributional concerns. Wildasin (1977) and Keen and Wildasin (2000) characterise the distortionary tax structures in each country which will result in globally Pareto-efficient outcomes, but their assumption of a single household in each country means that distributional considerations are not reflected in these domestic tax structures.

The first objective of this paper is therefore to characterise Pareto-efficient outcomes for a two-country world in which each country uses distortionary taxes both to raise revenue for public good provision and to redistribute between households within the country. Since these features of tax systems are observed in all countries, a characterisation of the domestic tax structures which are consistent with global Pareto-efficiency in such circumstances is an essential component of an analysis of whether domestic tax policy is being used for protectionist purposes.

The second objective of the paper is then to consider the problems involved in implementing a set of domestic tax structures which yield a Pareto-efficient outcome for the world economy when each country has incentives to use domestic tax policy to alter the terms of trade in its favour. As is well known, in the absence of restrictions on individual countries' use of tax policy to influence their terms of trade, the international trade equilibrium will not be Pareto-efficient. Our analysis of the domestic tax policies that countries will adopt in these circumstances builds upon, and extends, the existing literature on optimal taxes and tariffs in a large open economy (Boadway et al. 1973, Dixit 1985). We emphasise that restrictions on the domestic tax treatment of traded goods alone will not in general be sufficient to achieve global Pareto-efficiency, because (for standard second-best reasons) the domestic tax treatment of non-traded goods can be used to offset partially the effects of such restrictions and thereby achieve a measure of protection. This point was made by Vandendorpe (1972), but has not been prominent in more recent discussions of the ways in which domestic tax policy can be used to achieve indirect protection. Furthermore, Vandendorpe's analysis was cast in a framework in which other distortionary taxes were not needed and distributional issues did not arise. Hence, as well as reiterating the importance of Vandendorpe's point, another contribution of this paper is to generalise his analysis to a more realistic setting.

The plan of the paper is as follows. Section 2 sets out a model of trade between two countries, in each of which distortionary taxation of both traded and non-traded goods is used to raise revenue and achieve redistributional objectives. Section 3 characterises Pareto-efficient allocations for such a two-country world. Section 4 then analyses each country's choice of taxes and subsidies when it acts non-cooperatively, taking the other country's policy choices as given, and shows that the resulting Nash equilibrium tax and subsidy choices are not Pareto-efficient. It is shown that imposing constraints on countries' choices of taxation and subsidisation of traded goods in an attempt to achieve a Pareto-efficient outcome will fail if countries are left with some degrees of freedom in their decisions about taxes on and subsidies to non-traded goods. The implications of this analysis for attempts to achieve a globally Pareto-efficient outcome are discussed in section 5. Section 6 concludes the paper.

## 2 The model

We consider a model of trade between two countries, home and foreign. In each country there are $N$ private goods, the first $T$ of which are traded, the remaining $N - T$ being nontraded. These private goods have a wide interpretation, encompassing both goods and factors. The usual convention is adopted, that a negative supply by a firm represents a demand for an input and a negative demand by a household represents a supply. Since only relative prices matter, good 1 (a traded good) is taken as numeraire, and (without loss of generality) assumed to be untaxed in both countries. Thus the vector of world prices is written as $(1, \mathbf{p}^w)$, where $\mathbf{p}^w$ denotes the world prices of goods $2, ..., T$.[1]

The vector of net imports of traded goods by the home country is $(n_1, \mathbf{n})$, where the first component refers to good 1 and $\mathbf{n}$ denotes net imports of goods $2, ..., T$. Positive components of this vector correspond to imported goods, and negative components to exported goods. Since the effects of trade taxes can be replicated by suitable domestic commodity taxes and producer subsidies, the paper assumes that explicit trade taxes are not possible, and concentrates on the implications of the fact that the effects of such taxes can be replicated by domestic taxes and subsidies. The home country imposes commodity taxes $\mathbf{t}$ on home household demand $\mathbf{x}$ for traded goods $2, ..., T$, as well as producer subsidies $\mathbf{s}$ on home private firm supply $\mathbf{y}$ of traded goods $2, ..., T$. The sign of the product $t_i x_i$ shows whether there is a commodity tax or subsidy on household demand for a particular good $i = 2, ..., T$, so that a tax is indicated by $t_i > 0$ if $x_i > 0$, and by $t_i < 0$ if $x_i < 0$. Similarly the sign of the product $s_i y_i$ shows if there is a producer subsidy or tax on private firm supply of a particular good. The vector of home country consumer prices for traded goods is thus $(1, \mathbf{q})$, where $\mathbf{q} = \mathbf{p}^w + \mathbf{t}$, and the vector of home country producer prices for traded goods is thus $(1, \mathbf{p})$, where $\mathbf{p} = \mathbf{p}^w + \mathbf{s}$. The vector of home country producer prices of non-traded goods $T+1, ..., N$ is $\mathbf{p}_n$, and home household demand $\mathbf{x}_n$ for these goods is subject to commodity taxes $\mathbf{t}_n$.[2] Hence the vector of home country consumer prices for non-traded goods is $\mathbf{q}_n = \mathbf{p}_n + \mathbf{t}_n$.

Private production in the home country is carried out by a single competitive firm which trades at producer prices $(1, \mathbf{p}, \mathbf{p}_n)$ and has a strictly convex production set. The private firm's profit function is $\pi(1, \mathbf{p}, \mathbf{p}_n)$ and its (vector-valued) supply function is $(y_1(1, \mathbf{p}, \mathbf{p}_n), \mathbf{y}(1, \mathbf{p}, \mathbf{p}_n), \mathbf{y}_n(1, \mathbf{p}, \mathbf{p}_n))$.

The vector of home country government supply of private goods is $(z_1, \mathbf{z}, \mathbf{z}_n)$, positive and

[1] All vectors are column vectors, with a prime indicating transposition.

[2] Note that linear taxes on goods supplied by households are included in $(\mathbf{t}, \mathbf{t}_n)$.

negative components of which correspond respectively to outputs and inputs. In addition, the home country government produces a (local) pure public good $g$ using private goods as inputs. The home country government imposes commodity taxes $(\mathbf{t}, \mathbf{t}_n)$ and pays production subsidies $\mathbf{s}$. It also pays a uniform lump-sum transfer $r$ to all households $h = 1, ..., H$, and taxes private sector profits $\pi$ at a rate of 100%.[3] The home country government transacts at world prices for traded goods and producer prices for non-traded goods,[4] so that its budget constraint is

$$\mathbf{t}.\mathbf{x} + \mathbf{t}_n.\mathbf{x}_n - \mathbf{s}.\mathbf{y} + z_1 + \mathbf{p}^w.\mathbf{z} + \mathbf{p}_n.\mathbf{z}_n + \pi - \sum_{h=1}^{H} r = 0 \tag{1}$$

Each of the $H$ households in the home country has a budget constraint $x_1^h + \mathbf{q}.\mathbf{x}^h + \mathbf{q}_n.\mathbf{x}_n^h = r$, and chooses a utility-maximising vector of demands for private goods subject to this budget constraint and the given quantity of the public good. The resulting (vector-valued) demand functions are $(x_1^h(\mathbf{q}, \mathbf{q}_n, r, g), \mathbf{x}^h(\mathbf{q}, \mathbf{q}_n, r, g), \mathbf{x}_n^h(\mathbf{q}, \mathbf{q}_n, r, g))$.[5] The home country aggregate demand function is

$$(x_1(\mathbf{q}, \mathbf{q}_n, r, g), \mathbf{x}(\mathbf{q}, \mathbf{q}_n, r, g), \mathbf{x}_n(\mathbf{q}, \mathbf{q}_n, r, g)) = \sum_{h=1}^{H} (x_1^h(\mathbf{q}, \mathbf{q}_n, r, g), \mathbf{x}^h(\mathbf{q}, \mathbf{q}_n, r, g), \mathbf{x}_n^h(\mathbf{q}, \mathbf{q}_n, r, g))$$

Household preferences are represented by the indirect utility functions $v^h(\mathbf{q}, \mathbf{q}_n, r, g)$. Social welfare in the home country is given by a Bergson-Samuelson social welfare function $w(..., v^h(\mathbf{q}, \mathbf{q}_n, r, g), ...)$.

Behaviour in the foreign country is modelled in the same way as in the home country, and upper-case symbols are used to indicate prices, quantities, and behavioural functions in the foreign

[3] The assumption that private sector profits are fully taxed is one way of implementing the standard simplifying assumption in the optimal tax literature that there are no pure profits in household budget constraints. An alternative assumption which achieves the same effect is that private sector production takes place under constant returns to scale.

[4] If the government transacts at prices other than world and producer prices respectively, netting out taxes and subsidies within the government sector still yields equation (1).

[5] Since the consumer and producer prices of good 1 are both normalised to 1, these prices are suppressed as arguments of behavioural functions in the remainder of the paper.

country. An equilibrium for the world economy requires the following conditions to be satisfied.

$$x_1(\mathbf{q}, \mathbf{q}_n, r, g) - y_1(\mathbf{p}, \mathbf{p}_n) - z_1 + X_1(\mathbf{Q}, \mathbf{Q}_n, R, G) - Y_1(\mathbf{P}, \mathbf{P}_n) - Z_1 = 0 \tag{2}$$

$$\mathbf{x}(\mathbf{q}, \mathbf{q}_n, r, g) - \mathbf{y}(\mathbf{p}, \mathbf{p}_n) - \mathbf{z} + \mathbf{X}(\mathbf{Q}, \mathbf{Q}_n, R, G) - \mathbf{Y}(\mathbf{P}, \mathbf{P}_n) - \mathbf{Z} = \mathbf{0} \tag{3}$$

$$\mathbf{x}_n(\mathbf{q}, \mathbf{q}_n, r, g) - \mathbf{y}_n(\mathbf{p}, \mathbf{p}_n) - \mathbf{z}_n = \mathbf{0} \tag{4}$$

$$\mathbf{X}_n(\mathbf{Q}, \mathbf{Q}_n, R, G) - \mathbf{Y}_n(\mathbf{P}, \mathbf{P}_n) - \mathbf{Z}_n = \mathbf{0} \tag{5}$$

$$x_1(\mathbf{q}, \mathbf{q}_n, r, g) - y_1(\mathbf{p}, \mathbf{p}_n) - z_1 + \mathbf{p}^w.(\mathbf{x}(\mathbf{q}, \mathbf{q}_n, r, g) - \mathbf{y}(\mathbf{p}, \mathbf{p}_n) - \mathbf{z}) = 0 \tag{6}$$

$$X_1(\mathbf{Q}, \mathbf{Q}_n, R, G) - y_1(\mathbf{P}, \mathbf{P}_n) - Z_1 + \mathbf{p}^w.(\mathbf{X}(\mathbf{Q}, \mathbf{Q}_n, R, G) - \mathbf{Y}(\mathbf{P}, \mathbf{P}_n) - \mathbf{Z}) = 0 \tag{7}$$

Equation (2) is the world equilibrium condition for traded good 1, and equation (3) is the world equilibrium condition for the other $T-1$ traded goods. Equations (4) and (5) are the equilibrium conditions for non-traded goods in the home and foreign country respectively.[6] Equations (6) and (7) are the balance of trade conditions for the home and foreign country respectively: they state that the aggregate value at world prices of each country's net imports is zero. By definition, the home country's net imports of traded goods are $(n_1, \mathbf{n}) \equiv (x_1 - y_1 - z_1, \mathbf{x} - \mathbf{y} - \mathbf{z})$, and similarly for the foreign country.

Equations (3), (6), and (7) imply equation (2), so that equilibrium in the world market for the numeraire good is ensured if the world markets for all other traded goods are in equilibrium, and both countries' trade is balanced. Hence satisfaction of equation (2) is not explicitly required. It is straightforward to show, using the aggregate budget constraint of households, $x_1 + \mathbf{q}.\mathbf{x} + \mathbf{q}_n.\mathbf{x}_n = \sum_{h=1}^{H} r$, and the relationships $\pi = y_1 + \mathbf{p}.\mathbf{y} + \mathbf{p}_n \mathbf{y}_n$, $\mathbf{q} = \mathbf{p}^w + \mathbf{t}$, $\mathbf{q}_n = \mathbf{p}_n + \mathbf{t}_n$, and $\mathbf{p} = \mathbf{p}^w + \mathbf{s}$, that satisfaction of the government budget constraint (1) in the home country is implied by the equilibrium conditions (4) and (6). Similarly (5) and (7) imply that the foreign country government budget constraint is satisfied. Thus imposition of the conditions (3)-(7) ensures equilibrium in the markets for all goods and satisfaction of the government budget constraint in both countries.

[6] $\mathbf{0}$ in equation (3) is a $T-1$ vector of zeroes, while $\mathbf{0}$ in equations (4) and (5) is a $N-T$ vector of zeroes.

## 3 Pareto-efficient taxation

A Pareto-efficient allocation for the world economy is a set of home and foreign country taxes and subsidies, together with a set of world prices, such that, for a given value of social welfare in the foreign country (denoted by $\overline{W}$), social welfare in the home country is maximised subject to the constraints of world market equilibrium (equations (3)-(7)). The control variables for this problem are taxes and subsidies in the two countries ($\mathbf{t}, \mathbf{t}_n, \mathbf{s}, r$ and $\mathbf{T}, \mathbf{T}_n, \mathbf{S}, R$ in the home and foreign countries respectively), together with producer prices of non-traded goods in the two countries ($\mathbf{p}_n$ and $\mathbf{P}_n$) and world prices ($\mathbf{p}^w$).[7]

Letting $v_i$, $i = 2, ...T$, be Lagrange multipliers on the constraints (3), $v_k$ and $V_k$, $k = T+1, ..., N$, be Lagrange multipliers on the constraints (4) and (5), $\mu$ and $M$ be Lagrange multipliers on constraints (6) and (7), and $\Lambda$ be the Lagrange multiplier on the constraint that social welfare in the foreign country equals $\overline{W}$, the Lagrangean for this problem is

$$\begin{aligned} L \;=\; & w(..., v^h(\mathbf{p}^w + \mathbf{t}, \mathbf{p}_n + \mathbf{t}_n, r, g), ...) \\ & -\sum_{i=2}^{T} v_i \{x_i(\mathbf{p}^w + \mathbf{t}, \mathbf{p}_n + \mathbf{t}_n, r, g) - y_i(\mathbf{p}^w + \mathbf{s}, \mathbf{p}_n) - z_i \\ & + X_i(\mathbf{p}^w + \mathbf{T}, \mathbf{P}_n + \mathbf{T}_n, R, G) - Y_i(\mathbf{p}^w + \mathbf{S}, \mathbf{P}_n) - Z_i\} \\ & - \sum_{k=T+1}^{N} v_k \{x_k(\mathbf{p}^w + \mathbf{t}, \mathbf{p}_n + \mathbf{t}_n, r, g) - y_k(\mathbf{p}^w + \mathbf{s}, \mathbf{p}_n) - z_k\} \\ & - \sum_{k=T+1}^{N} V_k \{X_k(\mathbf{p}^w + \mathbf{T}, \mathbf{P}_n + \mathbf{T}_n, R, G) - Y_k(\mathbf{p}^w + \mathbf{S}, \mathbf{P}_n) - Z_k\} \\ & -\mu\{x_1(\mathbf{p}^w + \mathbf{t}, \mathbf{p}_n + \mathbf{t}_n, r, g) - y_1(\mathbf{p}^w + \mathbf{s}, \mathbf{p}_n) - z_1 \\ & + \sum_{i=2}^{T} p_i^w [x_i(\mathbf{p}^w + \mathbf{t}, \mathbf{p}_n + \mathbf{t}_n, r, g) - y_i(\mathbf{p}^w + \mathbf{s}, \mathbf{p}_n) - z_i]\} \\ & -M\{X_1(\mathbf{p}^w + \mathbf{T}, \mathbf{P}_n + \mathbf{T}_n, R, G) - Y_1(\mathbf{p}^w + \mathbf{S}, \mathbf{P}_n) - Z_1 \\ & + \sum_{i=2}^{T} p_i^w [X_i(\mathbf{p}^w + \mathbf{T}, \mathbf{P}_n + \mathbf{T}_n, R, G) - Y_i(\mathbf{p}^w + \mathbf{S}, \mathbf{P}_n) - Z_i]\} \\ & +\Lambda\{W[..., V^h(\mathbf{p}^w + \mathbf{T}, \mathbf{P}_n + \mathbf{T}_n, R, G), ...] - \overline{W}\} \end{aligned} \tag{8}$$

[7] Since our concern here is to characterise domestic tax structures which are Pareto-efficient for the world economy, we treat government production of the public good and government supply of private goods in the two countries as parameters.

The first-order necessary conditions obtained from this Lagrangean enable a globally Pareto-efficient allocation to be characterised. Since the details of this derivation are straightforward, they are relegated to Appendix A.

The first set of conditions required for Pareto-efficiency in the world economy concerns the relationship between shadow and producer prices. It is shown in the Appendix that it is possible to normalise $\mu = M = 1$. Hence, using the envelope theorem in (8), the shadow prices of traded goods $i = 2, ..., T$ in both the home and the foreign country at a globally Pareto-efficient allocation are given by $v_i + p_i^w$. Similarly, the shadow prices of non-traded goods $k = T+1, ..., N$ in the home country at such an allocation are given by $v_k$, while those for the foreign country are given by $V_k$. From (A14) and (A15) in Appendix A,

$$v_i + p_i^w = p_i \qquad i = 2, ..., T \tag{9}$$

$$v_k = p_k \qquad k = T+1, ..., N \tag{10}$$

$$v_i + p_i^w = P_i \qquad i = 2, ..., T \tag{11}$$

$$V_k = P_k \qquad k = T+1, ..., N \tag{12}$$

Equations (9) and (11) imply that, at a globally Pareto-efficient allocation, the producer prices of traded goods are the same in both countries, and, furthermore, that the shadow prices of traded goods in both countries are equal to the common producer prices. The former implication means that Pareto-efficiency for the world economy requires production efficiency in the use and production of traded goods.[8] Equations (10) and (12) imply that, at a globally Pareto-efficient allocation, in each country the producer prices of non-traded goods are equal to the shadow prices of non-traded goods.

[8] Keen and Wildasin (2000) point out that, if the number of traded goods is less than the number of countries, production efficiency is sufficient but not necessary for Pareto-efficiency for the world economy. However, if the number of traded goods is at least as large as the number of countries, production efficiency is necessary and sufficient for Pareto-efficiency for the world economy. Since our analysis assumes only two countries, it applies to the case where the number of traded goods is at least as large as the number of countries, which we regard as the empirically relevant one.

The second set of conditions required for global Pareto-efficiency concerns the relationship between consumer and producer prices, and the optimal choice of the uniform lump-sum transfer, in each country. These conditions characterise the choice of linear income tax and commodity taxes in each country at a globally Pareto-efficient allocation. From (A19) of Appendix A, the condition for the home country's optimal choice of uniform lump-sum transfer is

$$\frac{\sum_{h=1}^{H} c^h}{H} = 1 \tag{13}$$

where $c^h \equiv \beta^h + \sum_{i=2}^{T} (t_i - s_i) \left(\partial x_i^h / \partial m^h\right) + \sum_{k=T+1}^{N} t_k \left(\partial x_k^h / \partial m^h\right)$ is the net marginal social valuation of income accruing to household $h$. From (A20) of the Appendix, the relationship between consumer and producer prices in the home country at a globally Pareto-efficient allocation is characterised by

$$\widehat{\mathbf{x}}_{\mathbf{q}} \begin{pmatrix} \mathbf{q} - \mathbf{p} \\ \mathbf{q}_n - \mathbf{p}_n \end{pmatrix} = \sum_h (\mathbf{c}^h - \mathbf{1}) \mathbf{x}^h \tag{14}$$

where $\widehat{\mathbf{x}}_{\mathbf{q}}$ is the $(N-1)\times$ $(N-1)$ matrix of price derivatives of aggregate compensated demand functions and $\sum_h (\mathbf{c}^h - \mathbf{1})\mathbf{x}^h$ is the $(N-1)\times 1$ vector with components $\sum_{h=1}^{H} (c^h - 1) x_j^h$, $j = 2, ..., N$. Equations (13) and (14) together characterise the home country's choice of linear income tax and commodity taxes at a Pareto-efficient allocation for the world economy, and show that at such an allocation the home country sets these taxes in a way which corresponds exactly to standard rules for optimal taxation when distributional objectives matter.[9]

A similar argument shows that the foreign country's choice of linear income tax and commodity taxes at a globally Pareto-efficient allocation can be characterised by

$$\frac{\sum_{h=1}^{H} C^h}{H} = 1 \tag{15}$$

$$\widehat{\mathbf{X}}_{\mathbf{Q}} \begin{pmatrix} \mathbf{Q} - \mathbf{P} \\ \mathbf{Q}_n - \mathbf{P}_n \end{pmatrix} = \sum_h (\mathbf{C}^h - \mathbf{1}) \mathbf{X}^h \tag{16}$$

where $C^h \equiv \Lambda B^h + \sum_{i=2}^{T} (T_i - S_i) \partial X_i^h / \partial M^h + \sum_{k=T+1}^{N} T_k \partial X_k^h / \partial M^h$.

[9] See Diamond and Mirrlees (1971), Diamond (1975), and Atkinson and Stiglitz (1980), Lecture 14-2.

Pareto-efficient taxation for the world economy therefore requires the following. First, producer prices for all goods should equal shadow prices in both countries, and for traded goods producer prices should be the same in both countries. This does not rule out subsidies to private production of traded goods in the two countries, but it does imply that any such subsidies should be the same in both countries, i.e. $s_i = S_i$, $i = 2, ..., T$. Consequently production efficiency is a necessary condition for global Pareto-efficiency, provided that the number of traded goods is at least as large as the number of countries (Keen and Wildasin 2000). This desirability of production efficiency implies that the destination principle for commodity taxation is preferable to the origin principle, and that the residence principle for capital income taxation is superior to the source principle (Keen 1993, Keen and Wildasin 2000). Second, given producer prices which satisfy the above requirements, each country's choice of linear income tax and the relationship between consumer and producer prices should be characterised by standard optimal tax rules, which reflect that country's own judgement about the appropriate equity-efficiency trade-off within the country. Hence destination-based commodity taxes which differ between countries according to differences in distributional judgements and demand behaviour are consistent with global Pareto-efficiency.

# 4 Nash equilibrium taxation

Having characterised Pareto-efficient taxation for the world economy, we now analyse the taxes and subsidies which will be chosen if each country acts in its own interests, taking various aspects of the other country's behaviour as given. We focus on the home country, and assume that it chooses its taxation policy to maximise its social welfare function taking as given the foreign country's taxes, subsidies, and producer prices of non-traded goods, as well as its net import functions $X_j(\mathbf{p}^w + \mathbf{T}, \mathbf{P}_n + \mathbf{T}_n, R, G) - Y_j(\mathbf{p}^w + \mathbf{S}, \mathbf{P}_n) - Z_j$, $j = 1, ..., T$, which must satisfy the foreign country's balanced trade constraint

$$X_1(\mathbf{p}^w + \mathbf{T}, \mathbf{P}_n + \mathbf{T}_n, R, G) - Y_1(\mathbf{p}^w + \mathbf{S}, \mathbf{P}_n) - Z_1 \\ + \sum_{i=2}^{T} p_i^w \{X_i(\mathbf{p}^w + \mathbf{T}, \mathbf{P}_n + \mathbf{T}_n, R, G) - Y_i(\mathbf{p}^w + \mathbf{S}, \mathbf{P}_n) - Z_i\} = 0 \quad (17)$$

The equilibrium conditions which constrain the home country's policy choice are equations (3), (4), and (6). These conditions ensure that the home country's government satisfies its budget constraint. It is also straightforward to show that (17) together with (3) and (6) ensure that the world market for good 1 is in equilibrium, so that this constraint does not have to be imposed explicitly.

We also assume that the home country faces binding constraints on its choices of taxes on and subsidies to traded goods, denoted respectively by $\overline{t}_i$ and $\overline{s}_i$, $i = 2, ..., T$. These constraints may be interpreted either as constraints imposed on the home country by a supranational body or, more plausibly, as constraints to which the home country has agreed as part of a negotiated international trade agreement. For the purposes of our analysis, the precise interpretation of these constraints does not matter. We assume that the constraints relate only to the domestic tax treatment of traded goods because we wish to emphasise how the taxation of non-traded goods may be used to achieve a measure of protection by partially offsetting the effects of restrictions on these constraints.

The home country thus maximises its social welfare function subject to these constraints, taking the foreign country's behaviour as given in the way described above. The control variables for the home country are all its taxes and subsidies, and its producer prices for non-traded goods, i.e., $\mathbf{t}, \mathbf{t}_n, \mathbf{s}, r$, and $\mathbf{p}_n$, together with world prices $\mathbf{p}^w$.[10] The Lagrangean for this problem is

$$
\begin{aligned}
L \;=\; & w(\ldots, v^h(\mathbf{p}^w + \mathbf{t}, \mathbf{p}_n + \mathbf{t}_n, r, g), \ldots) \\
& - \sum_{i=2}^{T} v_i \{x_i(\mathbf{p}^w + \mathbf{t}, \mathbf{p}_n + \mathbf{t}_n, r, g) - y_i(\mathbf{p}^w + \mathbf{s}, \mathbf{p}_n) - z_i \\
& + X_i(\mathbf{p}^w + \mathbf{T}, \mathbf{P}_n + \mathbf{T}_n, R, G) - Y_i(\mathbf{p}^w + \mathbf{S}, \mathbf{P}_n) - Z_i\} \\
& - \sum_{k=T+1}^{N} v_k \{x_k(\mathbf{p}^w + \mathbf{t}, \mathbf{p}_n + \mathbf{t}_n, r, g) - y_k(\mathbf{p}^w + \mathbf{s}, \mathbf{p}_n) - z_k\} \\
& - \mu\{x_1(\mathbf{p}^w + \mathbf{t}, \mathbf{p}_n + \mathbf{t}_n, r, g) - y_1(\mathbf{p}^w + \mathbf{s}, \mathbf{p}_n) - z_1 \\
& + \sum_{i=2}^{T} p_i^w [x_i(\mathbf{p}^w + \mathbf{t}, \mathbf{p}_n + \mathbf{t}_n, r, g) - y_i(\mathbf{p}^w + \mathbf{s}, \mathbf{p}_n) - z_i]\} \\
& + \sum_{i=2}^{T} \gamma_i \left(\overline{s}_i - s_i\right) + \sum_{i=2}^{T} \delta_i \left(\overline{t}_i - t_i\right)
\end{aligned}
\tag{18}
$$

where $\gamma_i$ and $\delta_i$, $i = 2, \ldots, T$, are the Lagrange multipliers on the constraints relating to taxes and subsidies on traded goods. For notational simplicity we continue to use the symbols $v_i$, $v_k$, and $\mu$ for the Lagrange multipliers on the constraints (3), (4), and (6) despite the fact that this maximisation problem differs from the one analysed in the previous section. This problem has $3(T-1) + 2(N-T) + 1$ control variables, and $3(T-1) + (N-T) + 1$ constraints, so that the number of degrees of freedom for the home country's policy choice (given the constraints on $\mathbf{t}$ and $\mathbf{s}$) is $N - T$. The first-order necessary conditions for a solution to this problem are derived in Appendix B.

## 4.1 No constraints on the domestic tax treatment of traded goods

Consider first the 'unconstrained' case in which there are no constraints on the home country's choice of taxes and production subsidies on traded goods (formally this is the case in which $\overline{\mathbf{t}}$ and $\overline{\mathbf{s}}$ take values such that $\boldsymbol{\gamma} = \boldsymbol{\delta} = \mathbf{0}$). In this case (B8) in Appendix B gives $p_i = v_i + p_i^w$, and thus $v_i = s_i$, $i = 2, \ldots, T$, and $p_k = v_k$, $k = T+1, \ldots, N$. Using (B1) and (B3) in (B6), recalling that

[10] For simplicity we do not treat government production of the public good and government supply of private goods as control variables, although, as we note in section 5, the home country will also wish to choose these variables to exploit its ability to influence its terms of trade.

$\mu = 1$ and $v_i = s_i$, $i = 2, ..., T$, and letting $n_j = x_j - y_j - z_j$, $j = 2, ..., T$, denote the home country's net imports of good $j$, gives

$$\sum_{i=2}^{T} s_i \left( \frac{\partial Y_i}{\partial P_j} - \frac{\partial X_i}{\partial Q_j} \right) - n_j = 0 \qquad j = 2, ..., T$$

or

$$(\mathbf{Y_P} - \mathbf{X_Q})(\mathbf{s}) = (\mathbf{n}) \tag{19}$$

where $(\mathbf{Y_P} - \mathbf{X_Q})$ is the $(N-1) \times (N-1)$ matrix of price derivatives of the foreign country's net export functions,[11] $(\mathbf{s})$ is the $(N-1) \times 1$ vector of home country production subsidies to traded goods, and $(\mathbf{n})$ is the $(N-1) \times 1$ vector of home country net imports. Assuming that $(\mathbf{Y_P} - \mathbf{X_Q})$ is invertible, (19) solves to characterise the home country's optimal production subsidies to traded goods in the unconstrained case as follows

$$(\mathbf{s}) = (\mathbf{Y_P} - \mathbf{X_Q})^{-1} (\mathbf{n}) \tag{20}$$

Equation (20) is a standard expression for optimal tariffs.[12] Producer prices for traded goods in the home country are in this case related to world prices by production subsidies set on standard optimal tariff criteria, so that producer prices for all goods can be written as

$$\begin{pmatrix} \mathbf{p} \\ \mathbf{p}_n \end{pmatrix} = \begin{pmatrix} \mathbf{p}^w + (\mathbf{Y_P} - \mathbf{X_Q})^{-1} (\mathbf{n}) \\ \mathbf{v}_n \end{pmatrix} \tag{21}$$

The same argument as that used in Appendix A to obtain (13) and (14) gives

$$\frac{\sum_{h=1}^{H} c^h}{H} = 1 \tag{22}$$

$$\widehat{\mathbf{x}}_{\mathbf{q}} \begin{pmatrix} \mathbf{q} - \mathbf{p} \\ \mathbf{q}_n - \mathbf{p}_n \end{pmatrix} = \sum_h (\mathbf{c}^h - \mathbf{1}) \mathbf{x}^h \tag{23}$$

as the characterisation of the home country's optimal choice of commodity taxes and linear income tax in the unconstrained case. In this case, standard optimal tariff considerations determine the

[11] The foreign country's net export functions are the negative of its net import functions $N_j = X_j(1, \mathbf{p}^w + \mathbf{T}, \mathbf{P}_n + \mathbf{T}_n, R, G) - Y_j(1, \mathbf{p}^w + \mathbf{S}, \mathbf{P}_n) - Z_j$, $j = 2, ..., T$.

[12] Compare, for example, Dixit and Norman (1980), equation (62), page 152.

relationship between producer prices and world prices for traded goods, and standard optimal tax considerations determine the relationship between consumer and producer prices for all goods.

A Nash equilibrium for the two-country world economy in the unconstrained case will be one in which the foreign country's choice of control variables is also characterised by equations which take the form of (21)-(23), and, for each country, the taxes, subsidies, producer prices, and net import functions of the other country which it takes as given when it makes its own best choices are best responses of the other country to these best choices. We assume that such a Nash equilibrium always exists.

In the unconstrained case, the only way in which the form of the rules characterising the two countries' Nash equilibrium choices of their control variables differ from those which characterise Pareto-efficiency for the world economy is in the relationship between producer prices for traded goods in the two countries. At a globally Pareto-efficient allocation, producer prices for traded goods must be the same in both countries (recall equations (9) and (11)). But (21) for the home country, and its analogue for the foreign country, mean that this condition will typically not be satisfied at a Nash equilibrium in the unconstrained case, although within each country the producer prices of all goods (traded and non-traded) equal the shadow prices of goods in that country. To see this as simply as possible, assume that there are only two traded goods (1 and 2). Then the producer price of good 2 in the home country is (from (21))

$$p_2 = p_2^w + n_2 / \left[ (\partial Y_2/\partial P_2) - (\partial X_2/\partial Q_2) \right]$$

while that in the foreign country is

$$P_2 = p_2^w + N_2 / \left[ (\partial y_2/\partial p_2) - (\partial x_2/\partial q_2) \right]$$

Assume that $[(\partial Y_2/\partial P_2) - (\partial X_2/\partial Q_2)] > 0$ and $[(\partial y_2/\partial p_2) - (\partial x_2/\partial q_2)] > 0$.[13] Without loss of generality, let the home country import good 2 at a Nash equilibrium. Then, since at a Nash

[13] A sufficient condition for the former is $\partial X_2/\partial Q_2 < 0$, and for the latter $\partial x_2/\partial q_2 < 0$.

equilibrium $n_2 + N_2 = 0$, the home country will pay a positive production subsidy to domestic private production of good 2, so that $p_2 > p_2^w$, while the foreign country will pay a negative production subsidy to domestic private production of good 2, so that $P_2 < p_2^w$. Producer prices for good 2 thus differ between the two countries, and the Nash equilibrium for the world economy is not Pareto-efficient.

### 4.2 Constraints on production subsidies to traded goods

Next consider the case in which there are no constraints on commodity taxes on traded goods, but production subsidies to traded goods are constrained to be equal in both the home and the foreign country, so that producer prices of traded goods are the same in both countries. If these constraints bind, as they will in general, the values of the Lagrange multipliers corresponding to these constraints will be non-zero. Hence at the Nash equilibrium the relationship between producer and shadow prices in the home country will (from (B8)) take the form

$$\begin{pmatrix} \mathbf{p} \\ \mathbf{p}_n \end{pmatrix} = \begin{pmatrix} \mathbf{v} + \mathbf{p}^w \\ \mathbf{v}_n \end{pmatrix} - (\mathbf{y}_\mathbf{p})^{-1} \begin{pmatrix} \gamma \\ \mathbf{0} \end{pmatrix} \tag{24}$$

where $\mathbf{y}_\mathbf{p}$ is the $(N-1) \times (N-1)$ matrix of price derivatives of private sector supply functions, which is assumed to be invertible.

It is clear from (24) that the constraints on the home country's choice of subsidies to traded goods will typically affect the relationship between producer and shadow prices for all goods. In the case with two traded goods (1 and 2) and one non-traded good (3), (24) gives

$$\begin{aligned} p_2 &= v_2 + p_2^w - (\gamma_2 \partial y_3 / \partial p_3) \, / \, |y_p| \\ p_3 &= v_3 + (\gamma_2 \partial y_2 / \partial p_3) \, / \, |y_p| \end{aligned}$$

where $|y_p|$ is the determinant of the matrix of price derivatives of private sector supply functions in the home country. By the theory of the competitive producer, $|y_p| \geq 0$, and for the purposes of this example we assume that the inequality is strict. Suppose that $n_2 > 0$ at the Nash equilibrium:

then the home country would, if possible, wish to increase its subsidy to private production of good 2, so that $\gamma_2 > 0$. In the Nash equilibrium, therefore, $p_2 < v_2 + p_2^w$ provided that $\partial y_3/\partial p_3 > 0$,[14] while $p_3 > v_3$ if $\partial y_2/\partial p_3 > 0$ and conversely if $\partial y_2/\partial p_3 < 0$. The shadow price of good 2 in the home country exceeds its producer price, because the constraint on the home country's choice of subsidy to private production of good 2 prevents it setting a subsidy high enough to align the producer price with the shadow price. If $\partial y_2/\partial p_3 > 0$, so that private production of good 2 is raised by an increase in the producer price of good 3, the producer price of good 3 exceeds its shadow price, because such an excess provides an indirect way of stimulating private production of good 2 in the presence of the constraint on a direct subsidy to private production of good 2. The converse applies if $\partial y_2/\partial p_3 < 0$.

Thus, although the constraints on production subsidies ensure that producer prices for traded goods are the same in both countries, the Nash equilibrium is not Pareto-efficient, because, in general, the producer prices for all goods differ from shadow prices. In addition, the constraints on production subsidies to traded goods will typically influence the home country's choice of commodity taxes and linear income tax in such a way that the necessary conditions for a Pareto-efficient allocation with respect to these taxes do not hold. We defer consideration of this point until after discussion of the case in which there are constraints on both production subsidies to and commodity taxes on traded goods.

### 4.3 Constraints on both taxation and subsidisation of traded goods

The final case we consider is that in which, as well as constraints on production subsidies to traded goods, there are binding constraints on commodity taxes on traded goods in the home country which prevent it using such taxes to offset the effects of the subsidy constraints. In this case $\boldsymbol{\gamma} \neq \mathbf{0}$ and $\boldsymbol{\delta} \neq \mathbf{0}$. Equation (24) still characterises the relationship between producer and shadow prices

[14] By the theory of the competitive producer, $\partial y_3/\partial p_3 \geq 0$.

in the home country. The same argument as that in Appendix A (using (A7), (A16), and (A17)) now gives

$$\sum_{h=1}^{H} b^h = 0 \tag{25}$$

$$\widehat{\mathbf{x}}_{\mathbf{q}} \begin{pmatrix} \mathbf{q}-(\mathbf{v}+\mathbf{p}^w) \\ \mathbf{q}_n-\mathbf{v}_n \end{pmatrix} = \sum_h \mathbf{b}^h\mathbf{x}^h + \begin{pmatrix} \boldsymbol{\delta} \\ \mathbf{0} \end{pmatrix} \tag{26}$$

as the characterisation of the home country's optimal linear income tax and commodity taxes, where $b^h \equiv \beta^h - \partial x_1^h/\partial m^h - \sum_{i=2}^{T}(v_i + p_i^w)\left(\partial x_i^h/\partial m^h\right) - \sum_{k=T+1}^{N} v_k \left(\partial x_k^h/\partial m^h\right)$. Using (24) and the definition of $b^h$, (25) and (26) can be written as

$$\frac{\sum_{h=1}^{H} d^h}{H} = 1 \tag{27}$$

$$\widehat{\mathbf{x}}_{\mathbf{q}} \begin{pmatrix} \mathbf{q}-\mathbf{p} \\ \mathbf{q}_n-\mathbf{p}_n \end{pmatrix} = \sum_h (\mathbf{d}^h - \mathbf{1})\mathbf{x}^h + \begin{pmatrix} \boldsymbol{\delta} \\ \mathbf{0} \end{pmatrix} + \widehat{\mathbf{x}}_{\mathbf{q}} \left(\mathbf{y}_{\mathbf{p}}\right)^{-1} \begin{pmatrix} \boldsymbol{\gamma} \\ \mathbf{0} \end{pmatrix} \tag{28}$$

Here

$$d^h \equiv \beta^h + \sum_{i=2}^{T} (t_i - s_i)\, \partial x_i^h/\partial m^h + \sum_{k=T+1}^{N} t_k \partial x_k^h/\partial m^h - \left(\frac{\partial \mathbf{x}^h}{\partial \mathbf{m}^h}\right)' \left(\mathbf{y}_{\mathbf{p}}\right)^{-1} \begin{pmatrix} \boldsymbol{\gamma} \\ \mathbf{0} \end{pmatrix} \tag{29}$$

where $\left(\frac{\partial \mathbf{x}^h}{\partial \mathbf{m}^h}\right)'$ is the $1 \times (N-1)$ vector with components $\partial x_j^h/\partial m^h$, $j = 2, ..., N$, and $\sum_h (\mathbf{d}^h - \mathbf{1})\mathbf{x}^h$ is the $(N-1) \times 1$ vector with components $\sum_{h=1}^{H}(d^h - 1)x_j^h$, $j = 2, ..., N$.

Equation (27) shows that the presence of constraints on production subsidies to traded goods alters the characterisation of the home country's optimal choice of uniform lump-sum transfer in the Nash equilibrium. Instead of the average value of $c^h$ being 1, the constraints on subsidies mean that this optimal choice is characterised by the average value of $d^h$ equalling 1. The difference between $c^h$ and $d^h$ is the term $-\left(\frac{\partial \mathbf{x}^h}{\partial \mathbf{m}^h}\right)' \left(\mathbf{y}_{\mathbf{p}}\right)^{-1} \binom{\boldsymbol{\gamma}}{\mathbf{0}}$, which reflects the difference (due to the constraints on subsidies) between shadow and producer prices of goods for which household $h$ alters its demand as a result of an increase in the uniform transfer.

Comparison of (28) with (23) shows several differences in the characterisation of the home country's optimal choices of commodity taxes in the Nash equilibrium when there are constraints on both subsidies to and taxes on traded goods. The right-hand side of (28) has two additional terms, reflecting the constraints on taxation and subsidisation of traded goods. In addition, the weights in the first term (the weighted sum of household demands for taxed goods) are the $d^h$ rather than the $c^h$ terms, so that the constraints on subsidies to traded goods also affect the weights used to measure the distributional significance of different taxed goods.

How will constraints on both commodity taxes on and production subsidies to traded goods affect the home country's optimal choices with respect to the producer prices and commodity taxation of non-traded goods? As before, the simplest possible example, with two traded goods (1 and 2) and one non-traded good (3), is used to illustrate the results. Suppose that $n_2 > 0$ at the Nash equilibrium, so that the home country would wish, if possible, to increase both its tax on demand for and its subsidy to private production of good 2, so that $\delta_2 > 0$ and $\gamma_2 > 0$. Equation (24) gives

$$p_3 = v_3 + (\gamma_2 \partial y_2 / \partial p_3) \,/\, |y_p|$$

as before. Suppose that $\partial y_2 / \partial p_3 < 0$: then the constraint on $s_2$ induces the home country to set $p_3$ below $v_3$ in order to increase private production of good 2. Equation (28) can be solved to give the difference between the consumer and producer price of the non-traded good in this example as

$$q_3 - p_3 = \frac{\sum_{h=1}^{H}(d^h - 1)\left[(\partial \widehat{x}_2/\partial q_2)\, x_3^h - (\partial \widehat{x}_2/\partial q_3)\, x_2^h\right]}{|\widehat{x}_q|} - \frac{\delta_2 \partial \widehat{x}_2/\partial q_3}{|\widehat{x}_q|} - \frac{\gamma_2 \partial y_2/\partial p_3}{|y_p|} \qquad (30)$$

where $|\widehat{x}_q|$ is the determinant of the matrix of price derivatives of aggregate compensated demand functions in the home country, and it is assumed that $|\widehat{x}_q| > 0$.[15] The first component on the right-hand side of (30) gives the difference between $q_3$ and $p_3$ due to standard optimal commodity tax considerations, where, however, the weights $d^h$ reflect the constraints on production subsidies to traded goods (recall (29)). The second component on the right-hand side of (30) gives the

[15] By the theory of the consumer, $|\widehat{x}_q| \geq 0$.

difference between $q_3$ and $p_3$ as a result of the constraints on the taxation of traded goods, while the third component reflects the effect of the constraints on subsidies to traded goods which is distinct from the effect operating via the weights $d^h$. Given that $\partial y_2/\partial p_3 < 0$, the third component is positive, so that $q_3 - p_3$ is larger than it would be otherwise on this account. The intuition is that $p_3$ is lower than $v_3$ in order to increase private production of good 2 given the constraint on $s_2$, but this reduction in $p_3$ is intended only to affect private production, not private consumption, and hence $q_3 - p_3$ is higher than otherwise in order to prevent the lower $p_3$ translating into a lower $q_3$. The second component takes the sign of $-\partial\widehat{x}_2/\partial q_3$, so that if $\partial\widehat{x}_2/\partial q_3 > 0$, this component makes $q_3 - p_3$ lower than it would be otherwise, and conversely if $\partial\widehat{x}_2/\partial q_3 < 0$. The intuition is that the home country wishes to lower aggregate compensated demand for the imported good, and, given the constraint on $t_2$, a lowering of $q_3$ relative to $p_3$ is an indirect way of doing so if $\partial\widehat{x}_2/\partial q_3 > 0$, and conversely if $\partial\widehat{x}_2/\partial q_3 < 0$.

It is clear that the Nash equilibrium when there are constraints on both the taxation and subsidisation of traded goods will not be Pareto-efficient. Not only will producer prices generally differ from shadow prices, but also the two countries' choices of linear income tax and commodity taxes will be characterised by conditions of the form (27) and (28) rather than the global Pareto-efficiency conditions (13)-(16).

For completeness, let us note that setting $\boldsymbol{\delta} = \mathbf{0}$ in (27) and (28) gives the conditions characterising the home country's choice of linear income tax and commodity taxes in the case where there are constraints only on production subsidies to traded goods. In this case, too, the constraints on production subsidies affect the form of these conditions in such a way that the necessary conditions for Pareto-efficiency are not satisfied in the Nash equilibrium. The right-hand side of (28) has two components when there are no constraints on the commodity taxes levied on traded goods. The first component is $\sum_h (\mathbf{d}^h - \mathbf{1})\mathbf{x}^h$, while the second reflects the effect of the constraints on subsidies to traded goods which is separate from that operating via the weights $d^h$. This second component is the one which has been analysed in the existing literature for the case when subsidies to traded goods are constrained to be zero and distortionary taxes do not have to be used for revenue raising. Friedlander and Vandendorpe (1968) and Keen (1989) focus on the use of commodity taxes for protection when all goods are tradeable, while Vandendorpe (1972) considers this issue when there are both traded and non-traded goods.[16] Our analysis incorporates these influences on the setting of optimal commodity taxes into a framework where distortionary taxes have to be used for redistribution and revenue-raising.

### 4.4 Summary

The general point which emerges from this analysis is that a country will typically have incentives to use taxes on and subsidies to non-traded goods to offset partially the effects of constraints relating to taxes on and subsidies to traded goods. Constraints imposed on individual countries' taxation and subsidisation of traded goods in order to prevent them from attempting to distort trade patterns to their advantage, and thus generating a Pareto-inefficient outcome for the world economy, will typically fail to achieve a globally Pareto-efficient outcome if countries are left with any degrees of freedom in their decisions about the taxation and subsidisation of non-traded goods.

## 5 Implementing a Pareto-efficient allocation for the world economy

In this section we consider how, when both countries act non-cooperatively, a Pareto-efficient allocation for the world economy can be achieved in principle, and the difficulties with achieving such an allocation in practice. It is clear from the analysis in the previous section that the imposition of constraints on the taxation and subsidisation of traded goods alone will not be sufficient to achieve a globally Pareto-efficient outcome. Some constraints on countries' taxation and subsidisation of non-traded goods are also required.

[16] Details of how the Friedlander and Vandendorpe, Keen, and Vandendorpe results can be obtained from (28) with $\boldsymbol{\delta} = \mathbf{0}$ are available from the authors on request.

In the framework of the previous section, the home country, when facing constraints $\overline{\mathbf{t}}$ and $\overline{\mathbf{s}}$ on its choices of taxes on and subsidies to traded goods, had $N - T$ degrees of freedom in its policy choice, and thus had scope to use its tax treatment of non-traded goods to offset the effects of these constraints. In order to implement a globally Pareto-efficient allocation, it is necessary to impose $N - T$ additional constraints on the home country, so that it has no degrees of freedom in its policy choice and thus has to make choices which are consistent with global Pareto-efficiency. Such an outcome can be achieved if the following constraints are imposed on the home country. First, its subsidies to the production of traded goods are the same as those on the foreign country, which gives a set of constraints $\overline{\mathbf{s}}$. Second, there are $N - T$ additional constraints $\overline{\mathbf{t}}_n$ on the home country's choice of taxes on non-traded goods, as well as those relating to the taxes on traded goods. Recalling the condition (14) for globally Pareto-efficient commodity taxes, and the fact that, by definition, $\mathbf{q} - \mathbf{p} = \mathbf{t} - \mathbf{s}$ and $\mathbf{q}_n - \mathbf{p}_n = \mathbf{t}_n$, the constraints $\overline{\mathbf{t}}$ and $\overline{\mathbf{t}}_n$ must satisfy

$$\widehat{\mathbf{x}}_{\mathbf{q}} \begin{pmatrix} \overline{\mathbf{t}} - \overline{\mathbf{s}} \\ \overline{\mathbf{t}}_n \end{pmatrix} = \sum_h (\mathbf{c}^h - \mathbf{1}) \mathbf{x}^h \tag{31}$$

where

$$c^h \equiv \beta^h + \sum_{i=2}^{T} \left( \overline{t}_i - \overline{s}_i \right) \left( \partial x_i^h / \partial m^h \right) + \sum_{k=T+1}^{N} \overline{t}_k \left( \partial x_k^h / \partial m^h \right) \tag{32}$$

Similar choices of the constraints $\overline{\mathbf{S}}$, $\overline{\mathbf{T}}$, and $\overline{\mathbf{T}}_n$ imposed on the foreign country, where these constraints must satisfy

$$\widehat{\mathbf{X}}_{\mathbf{Q}} \begin{pmatrix} \overline{\mathbf{T}} - \overline{\mathbf{S}} \\ \overline{\mathbf{T}}_n \end{pmatrix} = \sum_h (\mathbf{C}^h - \mathbf{1}) \mathbf{X}^h \tag{33}$$

and

$$C^h \equiv \Lambda B^h + \sum_{i=2}^{T} \left( \overline{T}_i - \overline{S}_i \right) \partial X_i^h / \partial M^h + \sum_{k=T+1}^{N} \overline{T}_k \partial X_k^h / \partial M^h \tag{34}$$

will, provided that $\overline{\mathbf{s}}$, $\overline{\mathbf{t}}$, $\overline{\mathbf{t}}_n$, $\overline{\mathbf{S}}$, $\overline{\mathbf{T}}$, and $\overline{\mathbf{T}}_n$ are consistent with the conditions required for equilibrium in traded and non-traded goods markets, and for balanced trade, ensure that the Nash equilibrium for the world economy is Pareto-efficient.

There are two main points to be made about the difficulties with achieving such an outcome in practice. Our discussion of these practical difficulties assumes that the constraints required to implement a globally Pareto-efficient allocation as a Nash equilibrium result from negotiations between countries as part of an international trade agreement: the problems would be similar, and probably greater, if the constraints were imposed by a supranational body. The first point is that very wide-ranging constraints are required to achieve a Pareto-efficient outcome. Each country's policy choice has to be restricted not only with respect to the taxation and subsidisation of traded goods, but also with respect to non-traded goods. The extremely wide range of constraints required is likely to cause problems by substantially increasing the complexity of the negotiations between countries about the details of the trade agreement. Furthermore, it must be noted that, in principle, the number of constraints required to achieve a Pareto-efficient outcome is unbounded. The argument above that Pareto-efficiency for the world economy can be achieved by imposing constraints on each country's taxes on non-traded goods as well as on the taxation and subsidisation of traded goods assumes that these constraints leave each country with no degrees of freedom in policy choice. However, countries will typically have other policy variables, thus giving them some further degrees of freedom, and in principle they will respond to the constraints imposed on them to achieve global Pareto-efficiency by altering their choices of the unconstrained policy variables in such a way that the resulting Nash equilibrium is Pareto-inefficient. In the model used in this paper, the quantity of the pure public good in each country ($g$ and $G$ respectively in the home and foreign countries) has been treated as a parameter. If instead $g$ and $G$ are treated as control variables, it is straightforward to show that global Pareto-efficiency will in general require constraints to be imposed on each country's choice of the public good as well as those on the taxation and subsidisation of traded and non-traded goods. Achievement of a Pareto-efficient allocation for the world economy as a Nash equilibrium requires enough constraints to ensure that there are no degrees of freedom in policy choice for any country: this implies a range and detail of negotiations that is practically infeasible.

The second point is that the information required to impose the constraints required for global Pareto-efficiency is enormous. The required constraints take a simple form for production subsidies to traded goods, but for commodity taxation they are very complicated. As is well-known, a large amount of information is required to implement optimal commodity tax rules. It is clear from (31) and (32) that the choice of $\overline{\mathbf{t}}$ and $\overline{\mathbf{t}}_n$, given $\overline{\mathbf{s}}$, requires detailed knowledge of the responses of aggregate compensated demands to prices and individual household demands to income, at prices and incomes which are likely to be substantially different from those observed. The constraints imposed on the countries must also, of course, be consistent with overall equilibrium in the world economy. An informational problem specific to the present context concerns the constraints which must be imposed on individual countries' commodity taxes in order to achieve Pareto-efficiency. As is clear from (31)-(34), there is absolutely no presumption that global Pareto-efficiency requires the same constraints to be imposed on the taxation of traded and non-traded goods in the home and foreign countries. In contrast to the constraints on production subsidies to traded goods, which simply require such subsidies to be equal in both countries, the constraints required on commodity taxation in the two countries will typically differ, both because the responses of aggregate compensated demands to prices and individual household demands to income will typically differ between the two countries, and because the required constraints depend on distributional judgements in the two countries (as represented by social marginal utilities of income), which will also typically differ between the two countries.

The information required to establish clearly whether differences between countries in commodity tax systems are consistent with global Pareto-efficiency or indirectly protectionist is very difficult to obtain. It may conceivably be possible to obtain unambiguous measures of the responses of aggregate compensated demands to prices and individual household demands to income in the two countries. Such measures would ensure that differences in the commodity tax constraints imposed on the two countries as a result of a negotiated trade agreement due to differences in demand behaviour were indeed justified, and thus consistent with global Pareto-efficiency. But it is very difficult indeed to see how this could be done for differences in distributional judgements between the two countries. Objective measures of country-specific distributional judgements are extremely elusive, and hence a country can always claim that what is actually a protectionist commodity tax structure is one which reflects its distributional objectives. The fact that globally Pareto-efficient commodity taxation for the two countries depends on their country-specific distributional judgements makes it almost impossible to prevent countries from using their commodity tax systems for protectionist purposes in the absence of objective measures of these judgements. This creates plenty of scope for conflict between individual countries in trade negotiations.

# 6 Conclusion

This paper has characterised the domestic tax systems which yield Pareto-efficient outcomes for the world economy when each country uses distortionary taxes for public good provision and redistribution between households, and has analysed the possibility of achieving such an outcome when countries choose their tax systems non-cooperatively while recognising their ability to influence the terms of trade. Pareto-efficiency for the world economy requires that relative producer prices should coincide with relative shadow prices for all goods in each country. Provided that the number of traded goods is at least as large as the number of countries, it also requires that relative producer prices for traded goods should be the same in all countries, so that production efficiency is a feature of such Pareto-efficient allocations. Global Pareto-efficiency is further characterised by each country choosing a linear income tax and taxes on commodities according to standard optimal commodity tax rules which reflect that country's own judgement about the appropriate equity-efficiency trade-off within it. However, if countries have incentives to use protectionist policies to improve their terms of trade, the implementation of a set of domestic tax structures which achieve a globally Pareto-efficient outcome as a Nash equilibrium is likely to prove very difficult indeed. Two considerations have been emphasised in arriving at this conclusion: first, the ability of countries to use tax policy with respect to non-traded goods for protectionist purposes, and second, the difficulty of establishing whether the commodity taxes imposed on final sales of goods to consumers in a country reflect that country's distributional judgements, in which case they are consistent with global Pareto-efficiency, or protectionism by that country, in which case they are not.

A more general implication of this paper is that, if countries impose externalities on each other through their choices of domestic tax policy, trade negotiations between them in an attempt to secure Pareto-improvements will inevitably involve a very wide range of considerations. In particular, it is clear from the preceding analysis that neither the claim that trade negotiations can be restricted solely to the domestic tax treatment of traded goods, nor the argument that distributional aspects of a country's tax system are a matter solely for that country and not the business of other countries, can be sustained.

Given the difficulties that would be involved in any attempt to implement a set of domestic tax structures which would achieve a Pareto-efficient outcome for the world economy, it seems likely that less ambitious policy objectives will have a greater chance of succeeding. An obvious route to explore is whether, starting from a set of domestic tax systems which are globally Pareto-inefficient, there exist relatively simple Pareto-improving multilateral reforms. A literature on this question does exist (Keen 1987, 1989, Turunen-Red and Woodland 1990, 1991, Lockwood 1997), but it does not take account of the two features which have been emphasised in our analysis: the fact that countries which wish to protect have incentives to respond to constraints on the tax treatment of some goods by altering the taxes on others, and the importance of distributional concerns in individual countries' tax systems. Incorporating these features into the analysis of Pareto-improving multilateral tax reforms for the world economy is an important task for future research.

## Appendices

## A Derivation of the conditions for Pareto-efficiency in the world economy

Defining $\beta^h \equiv (\partial w/\partial v^h)(\partial v^h/\partial m^h)$ and $B^h \equiv (\partial W/\partial V^h)(\partial V^h/\partial M^h)$ as the social marginal utility of income accruing to household $h$ in the home and foreign country respectively, where $m^h$ denotes the income of household $h$ in the home country and $M^h$ the income of household $h$ in the foreign country,[17] the Lagrangean (8) gives the following first-order necessary conditions with respect to the control variables

$$t_j : -\sum_{h=1}^{H} \beta^h x_j^h - \mu \frac{\partial x_1}{\partial q_j} - \sum_{i=2}^{T} (v_i + \mu p_i^w) \frac{\partial x_i}{\partial q_j} - \sum_{k=T+1}^{N} v_k \frac{\partial x_k}{\partial q_j} = 0 \quad j = 2, ..., N \tag{A1}$$

$$T_j : -\Lambda \sum_{h=1}^{H} B^h X_j^h - M \frac{\partial X_1}{\partial Q_j} - \sum_{i=2}^{T} (v_i + M p_i^w) \frac{\partial X_i}{\partial Q_j} - \sum_{k=T+1}^{N} V_k \frac{\partial X_k}{\partial Q_j} = 0 \quad j = 2, ..., N \tag{A2}$$

$$s_j : \mu \frac{\partial y_1}{\partial p_j} + \sum_{i=2}^{T} (v_i + \mu p_i^w) \frac{\partial y_i}{\partial p_j} + \sum_{k=T+1}^{N} v_k \frac{\partial y_k}{\partial p_j} = 0 \quad j = 2, ..., T \tag{A3}$$

$$S_j : M \frac{\partial Y_1}{\partial P_j} + \sum_{i=2}^{T} (v_i + M p_i^w) \frac{\partial Y_i}{\partial P_j} + \sum_{k=T+1}^{N} V_k \frac{\partial Y_k}{\partial P_j} = 0 \quad j = 2, ..., T \tag{A4}$$

$$\begin{aligned} p_j : & -\sum_{h=1}^{H} \beta^h x_j^h - \mu \left( \frac{\partial x_1}{\partial q_j} - \frac{\partial y_1}{\partial p_j} \right) - \sum_{i=2}^{T} (v_i + \mu p_i^w) \left( \frac{\partial x_i}{\partial q_j} - \frac{\partial y_i}{\partial p_j} \right) \\ & - \sum_{k=T+1}^{N} v_k \left( \frac{\partial x_k}{\partial q_j} - \frac{\partial y_k}{\partial p_j} \right) = 0 \quad j = T+1, ..., N \end{aligned} \tag{A5}$$

$$\begin{aligned} P_j : & -\Lambda \sum_{h=1}^{H} B^h X_j^h - M \left( \frac{\partial X_1}{\partial Q_j} - \frac{\partial Y_1}{\partial P_j} \right) - \sum_{i=2}^{T} (v_i + M p_i^w) \left( \frac{\partial X_i}{\partial Q_j} - \frac{\partial Y_i}{\partial P_j} \right) \\ & - \sum_{k=T+1}^{N} V_k \left( \frac{\partial X_k}{\partial Q_j} - \frac{\partial Y_k}{\partial P_j} \right) = 0 \quad j = T+1, ..., N \end{aligned} \tag{A6}$$

[17] For notational simplicity we assume that the number of households is the same in both countries.

$$r : \sum_{h=1}^{H} \beta^h - \mu \sum_{h=1}^{H} \frac{\partial x_1^h}{\partial m^h} - \sum_{i=2}^{T} (v_i + \mu p_i^w) \sum_{h=1}^{H} \frac{\partial x_i^h}{\partial m^h} - \sum_{k=T+1}^{N} v_k \sum_{h=1}^{H} \frac{\partial x_k^h}{\partial m^h} = 0 \tag{A7}$$

$$R : \Lambda \sum_{h=1}^{H} B^h - M \sum_{h=1}^{H} \frac{\partial X_1^h}{\partial M^h} - \sum_{i=2}^{T} (v_i + M p_i^w) \sum_{h=1}^{H} \frac{\partial X_i^h}{\partial M^h} - \sum_{k=T+1}^{N} V_k \sum_{h=1}^{H} \frac{\partial X_k^h}{\partial M^h} = 0 \tag{A8}$$

$$\begin{aligned} p_j^w : & - \sum_{h=1}^{H} \beta^h x_j^h - \mu \left( \frac{\partial x_1}{\partial q_j} - \frac{\partial y_1}{\partial p_j} \right) - \sum_{i=2}^{T} (v_i + \mu p_i^w) \left( \frac{\partial x_i}{\partial q_j} - \frac{\partial y_i}{\partial p_j} \right) \\ & - M \left( \frac{\partial X_1}{\partial Q_j} - \frac{\partial Y_1}{\partial P_j} \right) - \sum_{i=2}^{T} (v_i + M p_i^w) \left( \frac{\partial X_i}{\partial Q_j} - \frac{\partial Y_i}{\partial P_j} \right) \\ & - \sum_{k=T+1}^{N} v_k \left( \frac{\partial x_k}{\partial q_j} - \frac{\partial y_k}{\partial p_j} \right) - \sum_{k=T+1}^{N} V_k \left( \frac{\partial X_k}{\partial Q_j} - \frac{\partial Y_k}{\partial P_j} \right) \\ & - \mu (x_j - y_j - z_j) - M (X_j - Y_j - Z_j) - \Lambda \sum_{h=1}^{H} B^h X_j^h = 0 \quad j = 2, ..., T \end{aligned} \tag{A9}$$

The remaining first-order necessary conditions for a solution are the various constraint equations.

Using conditions (A1) and (A2) for $j = 2, ..., T$ and conditions (A3) and (A4) in (A9) gives (recalling the definition of net imports for each country) $\mu n_j + M N_j = 0$, $j = 2, ..., T$, or

$$(\mathbf{n} \;\; \mathbf{N}) \begin{pmatrix} \mu \\ M \end{pmatrix} = \mathbf{0} \tag{A10}$$

where $\mathbf{n}$ is the vector of the home country's, and $\mathbf{N}$ the vector of the foreign country's, net imports of goods $2, ..., T$. The components of each row of the matrix $(\mathbf{n} \;\; \mathbf{N})$ sum to zero, since the world markets for traded goods $2, ..., T$ are in equilibrium, and hence the rank of this matrix is less than two. Therefore (A10) has a nontrivial solution for $\mu$ and $M$. Provided that there is some trade between the two countries, the rank of $(\mathbf{n} \;\; \mathbf{N})$ is 1, and hence a unique solution (up to a factor of proportionality) exists for $\mu$ and $M$. Since the components of each row of $(\mathbf{n} \;\; \mathbf{N})$ sum to zero, this solution is $\mu = M$. From (8), $\mu$ and $M$ are the shadow prices of good 1 at a Pareto-efficient allocation in the home and foreign country respectively. Since only relative shadow prices matter, it is possible to normalise such that $\mu = M = 1$.

## A.1 Production efficiency

Setting $\mu = 1$, condition (A1) for $j = T+1, ..., N$ in (A5) gives, together with (A3)

$$\frac{\partial y_1}{\partial p_j} + \sum_{i=2}^{T}(v_i + p_i^w)\frac{\partial y_i}{\partial p_j} + \sum_{k=T+1}^{N} v_k \frac{\partial y_k}{\partial p_j} = 0 \quad j = 2, ..., N \tag{A11}$$

By the theory of the competitive producer

$$\frac{\partial y_1}{\partial p_j} + \sum_{i=2}^{T} p_i \frac{\partial y_i}{\partial p_j} + \sum_{k=T+1}^{N} p_k \frac{\partial y_k}{\partial p_j} = 0 \quad j = 2, ..., N \tag{A12}$$

Subtracting (A12) from (A11) gives

$$\sum_{i=2}^{T}(v_i + p_i^w - p_i)\frac{\partial y_i}{\partial p_j} + \sum_{k=T+1}^{N}(v_k - p_k)\frac{\partial y_k}{\partial p_j} = 0 \quad j = 2, ..., N$$

or

$$\mathbf{y_p}\begin{pmatrix} \mathbf{v} + \mathbf{p}^w - \mathbf{p} \\ \mathbf{v}_n - \mathbf{p}_n \end{pmatrix} = \mathbf{0} \tag{A13}$$

where $\mathbf{y_p}$ is the $(N-1) \times (N-1)$ matrix of price derivatives of private sector supply functions in the home country, $(\mathbf{v} + \mathbf{p}^w - \mathbf{p})$ is the $(T-1) \times 1$ subvector of differences between shadow and producer prices of traded goods in the home country, and $(\mathbf{v}_n - \mathbf{p}_n)$ is the $(N-T) \times 1$ subvector of differences between shadow and producer prices of non-traded goods in the home country. Assuming $\mathbf{y_p}$ is nonsingular, (A13) solves to give

$$\begin{pmatrix} \mathbf{p} \\ \mathbf{p}_n \end{pmatrix} = \begin{pmatrix} \mathbf{v} + \mathbf{p}^w \\ \mathbf{v}_n \end{pmatrix} \tag{A14}$$

A similar argument for the foreign country, using (A2), (A4), and (A6), gives

$$\begin{pmatrix} \mathbf{P} \\ \mathbf{P}_n \end{pmatrix} = \begin{pmatrix} \mathbf{v} + \mathbf{p}^w \\ \mathbf{V}_n \end{pmatrix} \tag{A15}$$

## A.2 Commodity taxes and linear income tax

To characterise Pareto-efficient commodity taxes in the home country, note that, using the Slutsky equation

$$\frac{\partial x_i}{\partial q_j} = \sum_{h=1}^{H}\left(\frac{\partial \widehat{x}_i^h}{\partial q_j} - x_j^h \frac{\partial x_i^h}{\partial m^h}\right) \qquad j = 2, ..., N$$

where $\widehat{x}_i^h$ denotes household $h$'s compensated demand for good $i$. Hence, recalling that $\mu = 1$, (A1) can be written as

$$-\sum_{h=1}^{H} \frac{\partial \widehat{x}_1^h}{\partial q_j} - \sum_{i=2}^{T} (v_i + p_i^w) \sum_{h=1}^{H} \frac{\partial \widehat{x}_i^h}{\partial q_j} - \sum_{k=T+1}^{N} v_k \sum_{h=1}^{H} \frac{\partial \widehat{x}_i^h}{\partial q_j} = \sum_{h=1}^{H} b^h x_j^h \quad j = 2, ..., N \tag{A16}$$

where $b^h \equiv \beta^h - \partial x_1^h / \partial m^h - \sum_{i=2}^{T} (v_i + p_i^w) \left( \partial x_i^h / \partial m^h \right) - \sum_{k=T+1}^{N} v_k \left( \partial x_k^h / \partial m^h \right)$ is the marginal social value of a transfer to household $h$, given by the social marginal utility of $h$'s income less the cost, at shadow prices, of meeting the additional demands resulting from such a transfer. Using the theory of the consumer, and aggregating over all households

$$\sum_{h=1}^{H} \frac{\partial \widehat{x}_1^h}{\partial q_j} + \sum_{i=2}^{T} q_i \sum_{h=1}^{H} \frac{\partial \widehat{x}_i^h}{\partial q_j} + \sum_{k=T+1}^{N} q_k \sum_{h=1}^{H} \frac{\partial \widehat{x}_k^h}{\partial q_j} = 0 \quad j = 2, ..., N \tag{A17}$$

Adding (A17) to (A16), and using (A14), gives

$$\sum_{i=2}^{T} (q_i - p_i) \sum_{h=1}^{H} \frac{\partial \widehat{x}_i^h}{\partial q_j} + \sum_{k=T+1}^{N} (q_k - p_k) \sum_{h=1}^{H} \frac{\partial \widehat{x}_k^h}{\partial q_j} = \sum_{h=1}^{H} b^h x_j^h \quad j = 2, ..., N$$

or

$$\widehat{\mathbf{x}}_{\mathbf{q}} \begin{pmatrix} \mathbf{q} - \mathbf{p} \\ \mathbf{q}_n - \mathbf{p}_n \end{pmatrix} = \sum_h \mathbf{b}^h \mathbf{x}^h \tag{A18}$$

where $\widehat{\mathbf{x}}_{\mathbf{q}}$ is the $(N-1) \times (N-1)$ matrix of price derivatives of aggregate compensated demand functions in the home country, $(\mathbf{q} - \mathbf{p})$ is the $(T-1) \times 1$ subvector of differences between consumer and producer prices of traded goods in the home country, $(\mathbf{q}_n - \mathbf{p}_n)$ is the $(N-T) \times 1$ subvector of differences between consumer and producer prices of non-traded goods in the home country, and $\sum_h \mathbf{b}^h \mathbf{x}^h$ is the $(N-1) \times 1$ vector with components $\sum_{h=1}^{H} b^h x_j^h$, $j = 2, ..., N$, i.e., the weighted sum of household demands for taxed goods, with the weights being marginal social values of transfers to households.

Equation (A7) characterises the home country's choice of uniform lump-sum transfer paid to all households at a Pareto-efficient allocation. Using $\mu = 1$, and recalling the definition of $b^h$, (A7) can be written $\sum_{h=1}^{H} b^h = 0$, which implies that this transfer is such that the sum over all households of the marginal social value of the transfer is zero.

Since, from (A14), producer prices equal shadow prices at a Pareto-efficient allocation, the marginal social value of a transfer to household $h$ can be written as $b^h = \beta^h - \partial x_1^h/\partial m^h - \sum_{i=2}^{T} p_i \left(\partial x_i^h/\partial m^h\right) - \sum_{k=T+1}^{N} p_k \left(\partial x_k^h/\partial m^h\right)$. Differentiating household $h$'s budget constraint $x_1^h + \sum_{i=2}^{T} (p_i + t_i - s_i)\, x_i^h + \sum_{k=T+1}^{N} (p_k + t_k)\, x_k^h = r$ with respect to $r$ gives

$$\partial x_1^h/\partial m^h + \sum_{i=2}^{T} p_i \left(\partial x_i^h/\partial m^h\right) + \sum_{k=T+1}^{N} p_k \left(\partial x_k^h/\partial m^h\right) = 1 - \sum_{i=2}^{T} (t_i - s_i) \left(\partial x_i^h/\partial m^h\right) - \sum_{k=T+1}^{N} t_k \left(\partial x_k^h/\partial m^h\right)$$

so that $b^h = \beta^h + \sum_{i=2}^{T} (t_i - s_i) \left(\partial x_i^h/\partial m^h\right) + \sum_{k=T+1}^{N} t_k \left(\partial x_k^h/\partial m^h\right) - 1$. Hence the condition for the optimal choice of uniform lump-sum transfer can be written as

$$\sum_{h=1}^{H} \left( \beta^h + \sum_{i=2}^{T} (t_i - s_i) \left(\partial x_i^h/\partial m^h\right) + \sum_{k=T+1}^{N} t_k \left(\partial x_k^h/\partial m^h\right) - 1 \right) = 0$$

or

$$\frac{\sum_{h=1}^{H} c^h}{H} = 1 \tag{A19}$$

where we define $c^h \equiv \beta^h + \sum_{i=2}^{T} (t_i - s_i) \left(\partial x_i^h/\partial m^h\right) + \sum_{k=T+1}^{N} t_k \left(\partial x_k^h/\partial m^h\right)$ as the net marginal social valuation of income accruing to household $h$. Similarly (A18) can be written as

$$\widehat{\mathbf{x}}_{\mathbf{q}} \begin{pmatrix} \mathbf{q} - \mathbf{p} \\ \mathbf{q}_n - \mathbf{p}_n \end{pmatrix} = \sum_h (\mathbf{c}^h - \mathbf{1}) \mathbf{x}^h \tag{A20}$$

where $\sum_h (\mathbf{c}^h - \mathbf{1}) \mathbf{x}^h$ is the $(N-1) \times 1$ vector with components $\sum_{h=1}^{H} (c^h - 1) x_j^h$, $j = 2, ..., N$.

# B Derivation of the Nash equilibrium conditions for the home country

The Lagrangean (18) gives the following first-order necessary conditions with respect to the control variables

$$t_j : -\sum_{h=1}^{H}\beta^h x_j^h - \mu\frac{\partial x_1}{\partial q_j} - \sum_{i=2}^{T}(v_i + \mu p_i^w)\frac{\partial x_i}{\partial q_j} - \sum_{k=T+1}^{N} v_k \frac{\partial x_k}{\partial q_j} - \delta_j = 0 \quad j = 2, ..., T \tag{B1}$$

$$t_j : -\sum_{h=1}^{H}\beta^h x_j^h - \mu\frac{\partial x_1}{\partial q_j} - \sum_{i=2}^{T}(v_i + \mu p_i^w)\frac{\partial x_i}{\partial q_j} - \sum_{k=T+1}^{N} v_k \frac{\partial x_k}{\partial q_j} = 0 \quad j = T+1, ..., N \tag{B2}$$

$$s_j : \mu\frac{\partial y_1}{\partial p_j} + \sum_{i=2}^{T}(v_i + \mu p_i^w)\frac{\partial y_i}{\partial p_j} + \sum_{k=T+1}^{N} v_k \frac{\partial y_k}{\partial p_j} - \gamma_j = 0 \quad j = 2, ..., T \tag{B3}$$

$$\begin{aligned} p_j : -\sum_{h=1}^{H}\beta^h x_j^h - \mu\left(\frac{\partial x_1}{\partial q_j} - \frac{\partial y_1}{\partial p_j}\right) - \sum_{i=2}^{T}(v_i + \mu p_i^w)\left(\frac{\partial x_i}{\partial q_j} - \frac{\partial y_i}{\partial p_j}\right) \\ - \sum_{k=T+1}^{N} v_k \left(\frac{\partial x_k}{\partial q_j} - \frac{\partial y_k}{\partial p_j}\right) = 0 \quad j = T+1, ..., N \end{aligned} \tag{B4}$$

$$r : \sum_{h=1}^{H}\beta^h - \mu\sum_{h=1}^{H}\frac{\partial x_1^h}{\partial m^h} - \sum_{i=2}^{T}(v_i + \mu p_i^w)\sum_{h=1}^{H}\frac{\partial x_i^h}{\partial m^h} - \sum_{k=T+1}^{N} v_k \sum_{h=1}^{H}\frac{\partial x_k^h}{\partial m^h} = 0 \tag{B5}$$

$$\begin{aligned} p_j^w : -\sum_{h=1}^{H}\beta^h x_j^h - \mu\left(\frac{\partial x_1}{\partial q_j} - \frac{\partial y_1}{\partial p_j}\right) - \sum_{i=2}^{T}(v_i + \mu p_i^w)\left(\frac{\partial x_i}{\partial q_j} - \frac{\partial y_i}{\partial p_j}\right) \\ -\sum_{i=2}^{T} v_i \left(\frac{\partial X_i}{\partial Q_j} - \frac{\partial Y_i}{\partial P_j}\right) - \sum_{k=T+1}^{N} v_k \left(\frac{\partial x_k}{\partial q_j} - \frac{\partial y_k}{\partial p_j}\right) - \mu(x_j - y_j - z_j) = 0 \quad j = 2, ..., T \end{aligned} \tag{B6}$$

The remaining first-order necessary conditions for a solution are the various constraint equations.

Since only relative shadow prices matter, $\mu$, which (from (18)) is the shadow price of good 1 in the home country, can be normalised to equal 1. Using the same argument as that leading to (A13) in Appendix A, it then follows that

$$\mathbf{y_p}\begin{pmatrix} \mathbf{v} + \mathbf{p}^w - \mathbf{p} \\ \mathbf{v}_n - \mathbf{p}_n \end{pmatrix} = \begin{pmatrix} \boldsymbol{\gamma} \\ \mathbf{0} \end{pmatrix} \tag{B7}$$

where $\boldsymbol{\gamma}$ is the $(T-1)\times 1$ subvector of solution values of the Lagrange multipliers on the constraints relating to production subsidies to traded goods, and $\mathbf{0}$ is a $(N-T)\times 1$ subvector of zeroes. Assuming that $\mathbf{y_p}$ is nonsingular, (B7) solves to give

$$\begin{pmatrix} \mathbf{v} + \mathbf{p}^w - \mathbf{p} \\ \mathbf{v}_n - \mathbf{p}_n \end{pmatrix} = (\mathbf{y_p})^{-1}\begin{pmatrix} \boldsymbol{\gamma} \\ \mathbf{0} \end{pmatrix} \tag{B8}$$

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